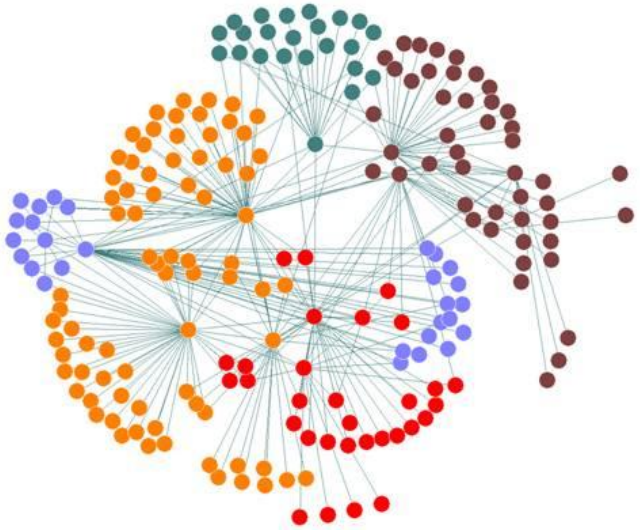
TEL AVIV UNIVERSITY

Algorithms and Applications in Social Networks



2025/2026, Semester A

Slava Novgorodov

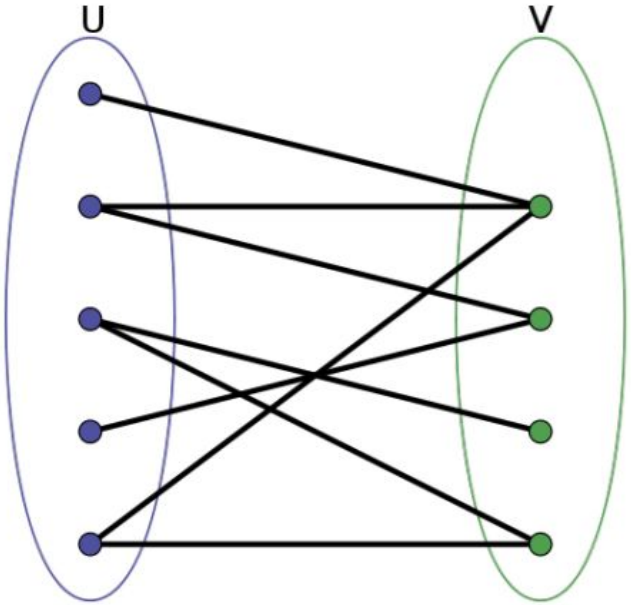
Lesson #3

- Bipartite graph (recap)
- Networks with Signed Edges
  - Single edge
  - Theory of Balance
  - Examples

Bipartite Graph

Bipartite Graph

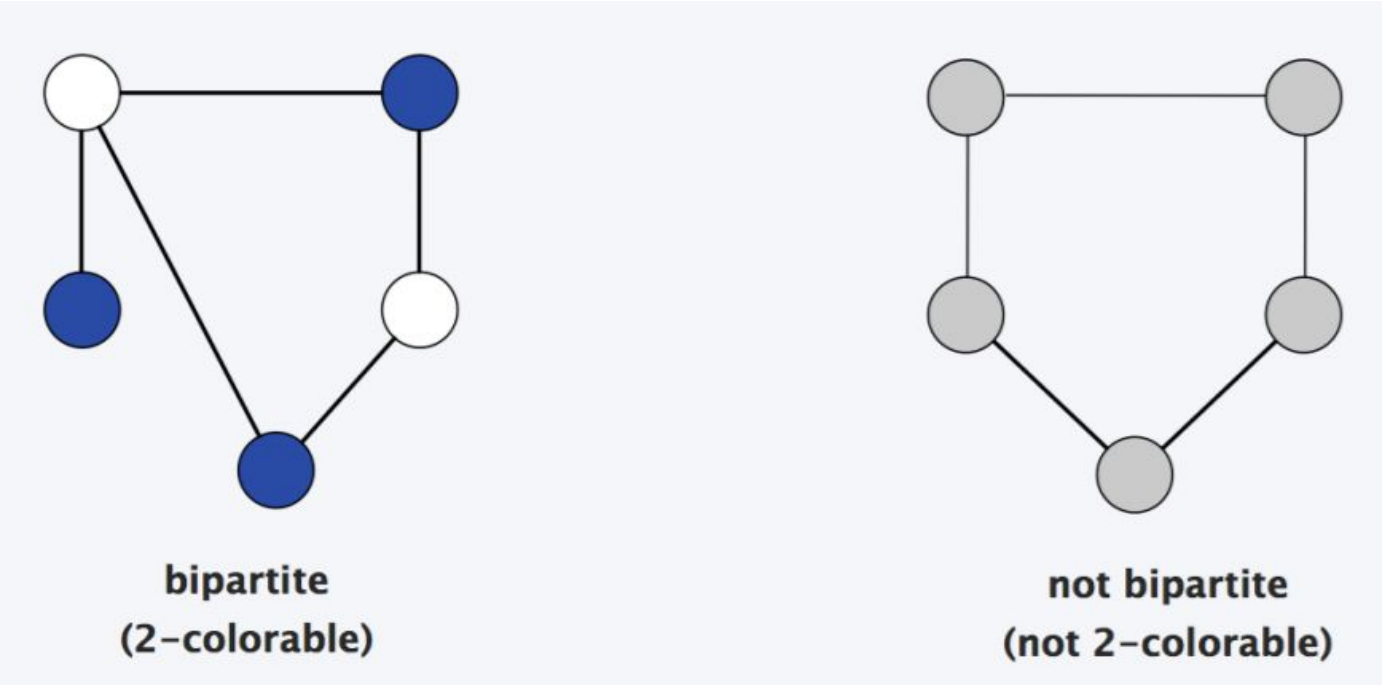
- A graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V



- A bipartite graph does not contain any odd-length cycles
- A bipartite graph can be vertex colored wtih 2 colors

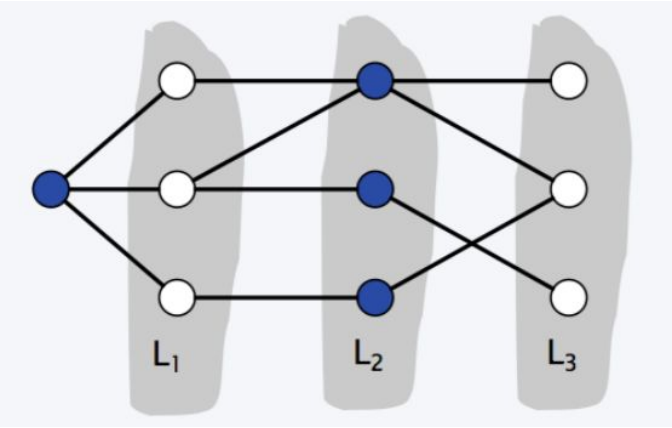
Testing Bipartiteness

- Triangle – not bipatite
- Graph contains an odd cycle – not bipartite



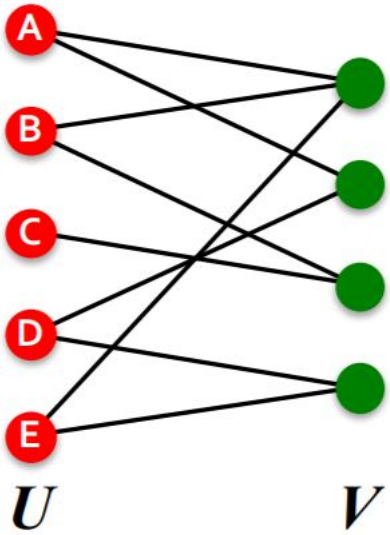
Testing Bipartiteness

- Is given graph bipartite?
- Algorithm:
  - Select and node and perform BFS, color each layer alternate colors
  - Scan all the edges, see if any edge has nodes with the same color (one layer nodes)

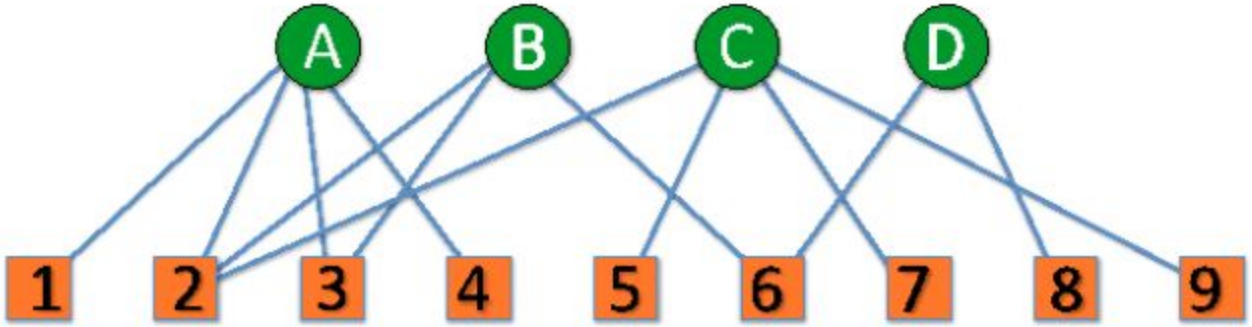


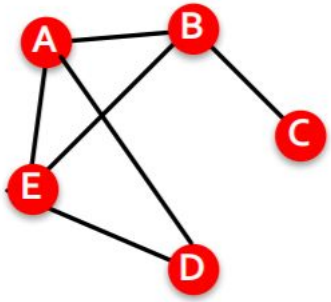
Usage of Bipartite Graph

- Different types of nodes:
  - Users/Items ranking
  - Papers/Authors
  - Courses/Students



Folded network:



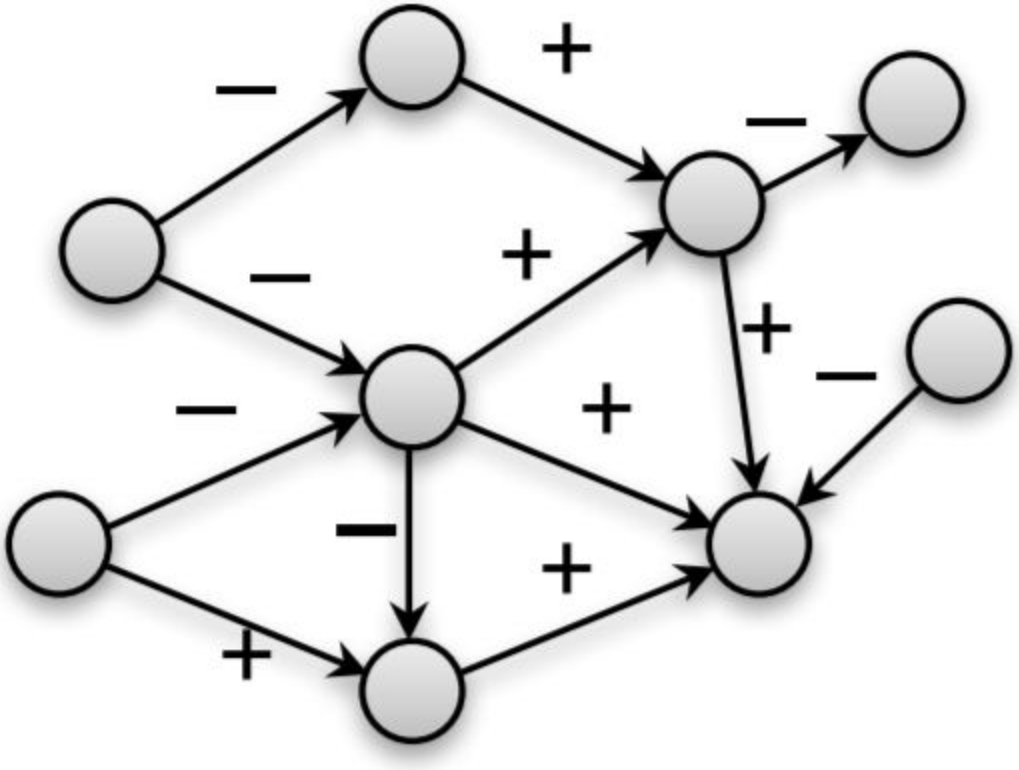


Folded version of the graph above

Networks with Signed Edges

Networks with Signed Edges

- Sometimes just “Signed Network”
- Can be directed or undirected



Motivation

- Trying to model behavior of people in (online) social networks
- Relationships between people in the network can be positive or negative
- People express opinion that can be positive or negative

Opinions of People in the Network

People can express opinion:

- By action:
  - Pressing “Like”/“Dislike” button
  - Giving rating to a product/person
- By writing text:
  - Comments, review, etc

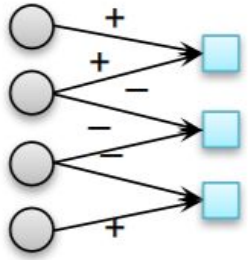
Applications:

- Recommendation systems
- Crowdsourcing

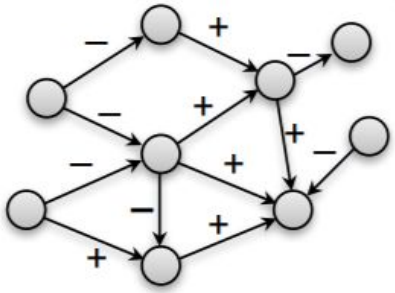
Types of Opinions

People can express opinion about:

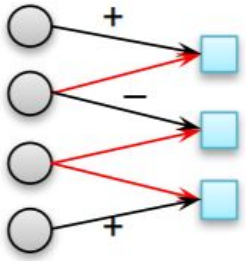
- Items:
  - Movies, hotels, purchases rating/reviews



- Other people:
  - GetTaxi drivers, AirBnB, Wikipedia



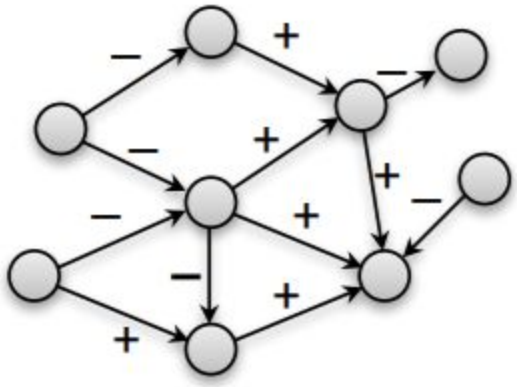
- Content generated by other people:
  - StackOverflow, Facebook



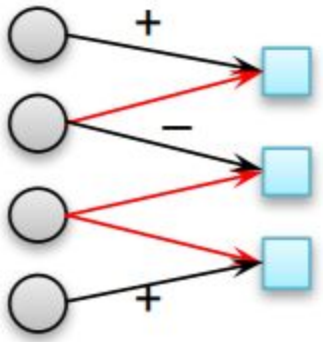
Evaluation

- Factors:
  - What drives people to give particular evaluation?

- Types: Direct/Indirect



Direct



Indirect

Datasets

Where it exists on the Web?

- Wikipedia moderators elections
  - Positive/Neagtive vote (120K votes in English)

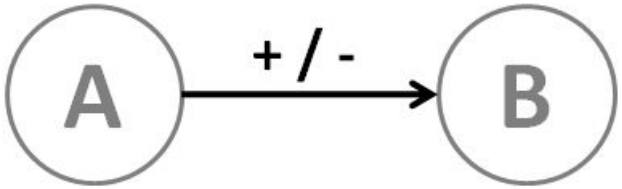
- StackOverflow Community
  - Upvotes/Downvotes (7.5M votes)

- Epinions product review
  - Ratings of product review (13M ratings)
  - 5 – positive, 1-4 – negative

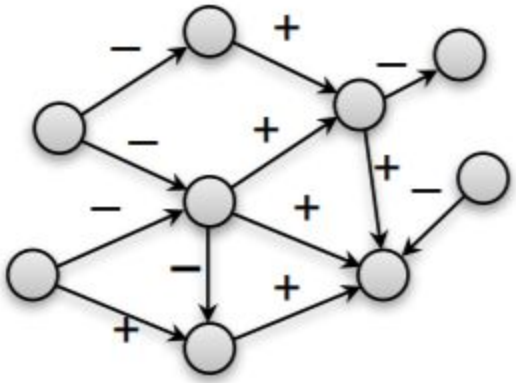
Evaluation – two ways to analyze

Two ways to look on it:

- Single evaluation (without network context)



- Evaluations in the context of the network

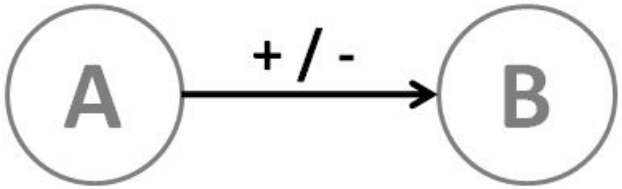


From: Effects of User Similarity in Social Media. A. Anderson, D. Huttenlocher, J. Kleinberg, J. Leskovec (WSDM 2012)

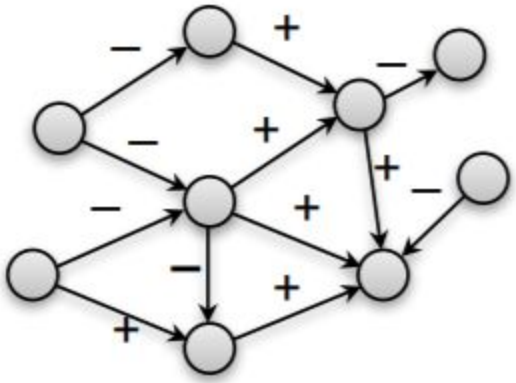
Evaluation – without context

Two ways to look on it:

- Single evaluation (without network context)

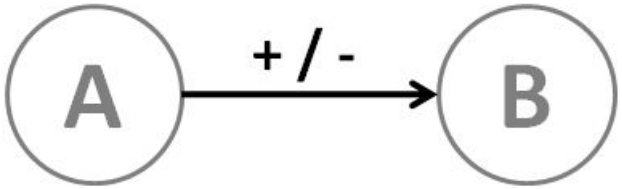


- Evaluations in the context of the network



Human Evaluation

- What drives human evaluation?

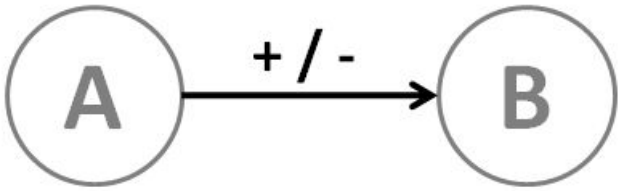


- Which (and whose) properties are important?
  - Properties of A?
  - Properties of B?
  - Which properties?

Important Properties

- **Status:**
  - Level of recognition, achievements, reputation in the community
    - Wikipedia: # of edits, # of new articles written
    - StackOverflow: # of answers
- **Similarity:**
  - Overlapping interests between A and B
    - Wikipedia: similarity of edited articles
    - StackOverflow: similarity of users evaluated

Relative vs. Absolute evaluation



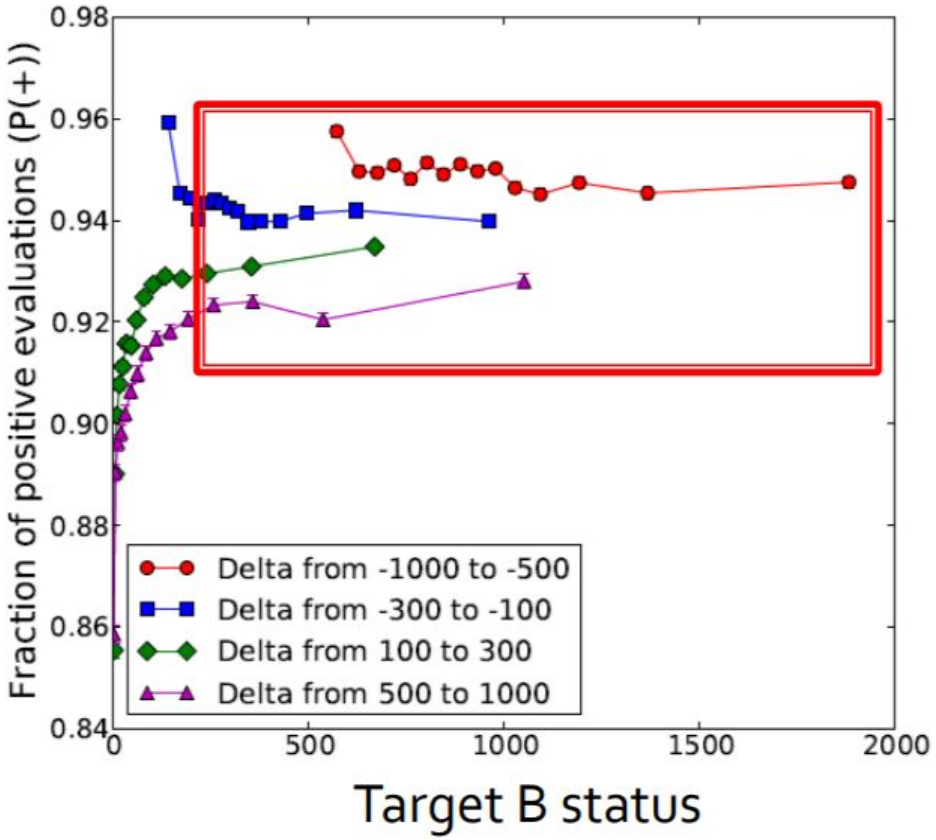
Two hypothesis:

- B receives a positive evaluation depends primarily on the characteristics of B
  - There is some objective criteria for user B to receive a positive evaluation
- B receives a positive evaluation depends on relationship between the characteristics of A and B
  - A compares herself to B

Effect of Status

- How does status of B affects A's evaluation?
- Status $\Delta = S_A - S_B$

- Observations:
  - P(+) doesn't depends on B's status
  - Different Δ implies different behavior



(Wikipedia dataset)

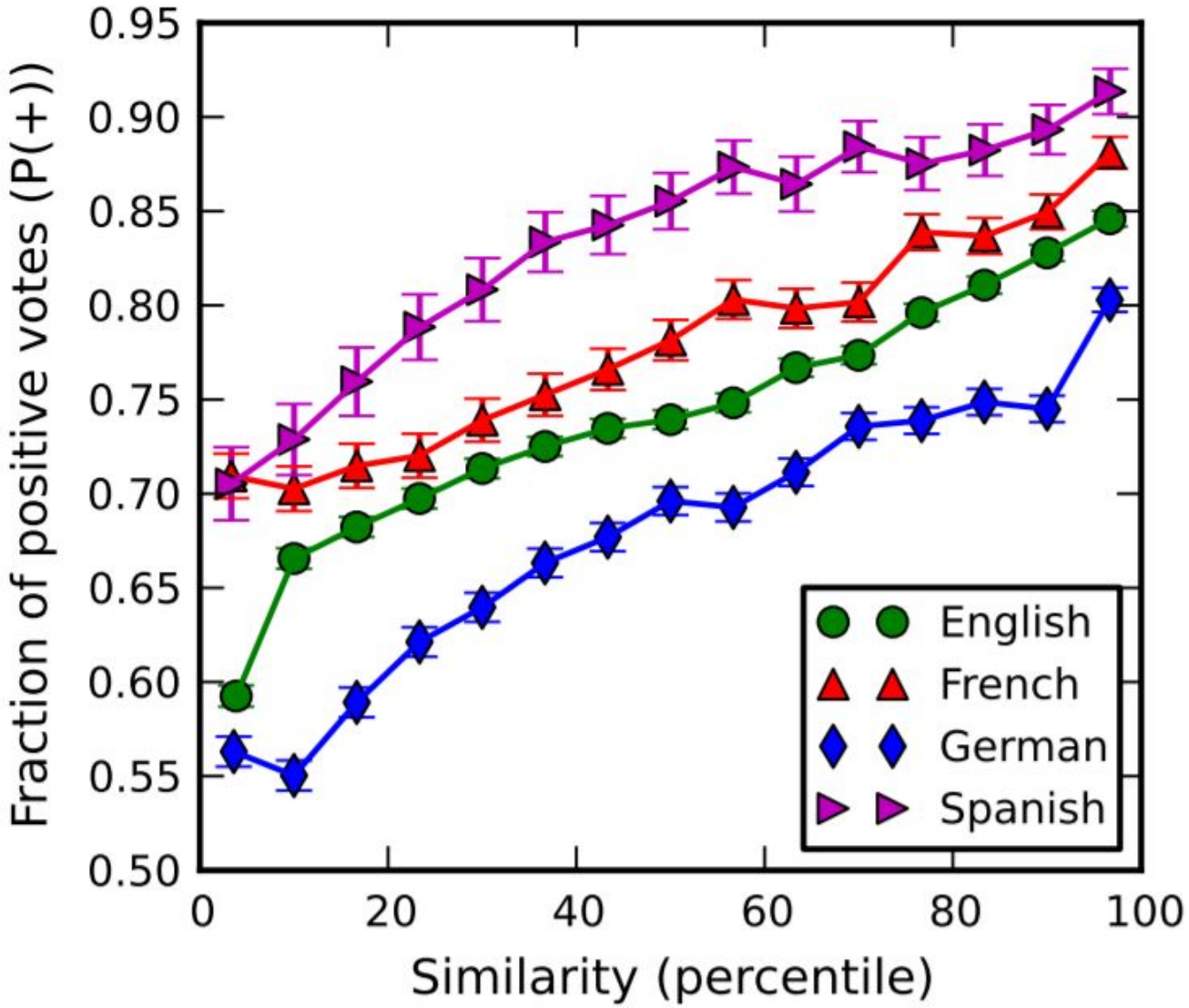
Effect of Similarity

Two hypothesis:

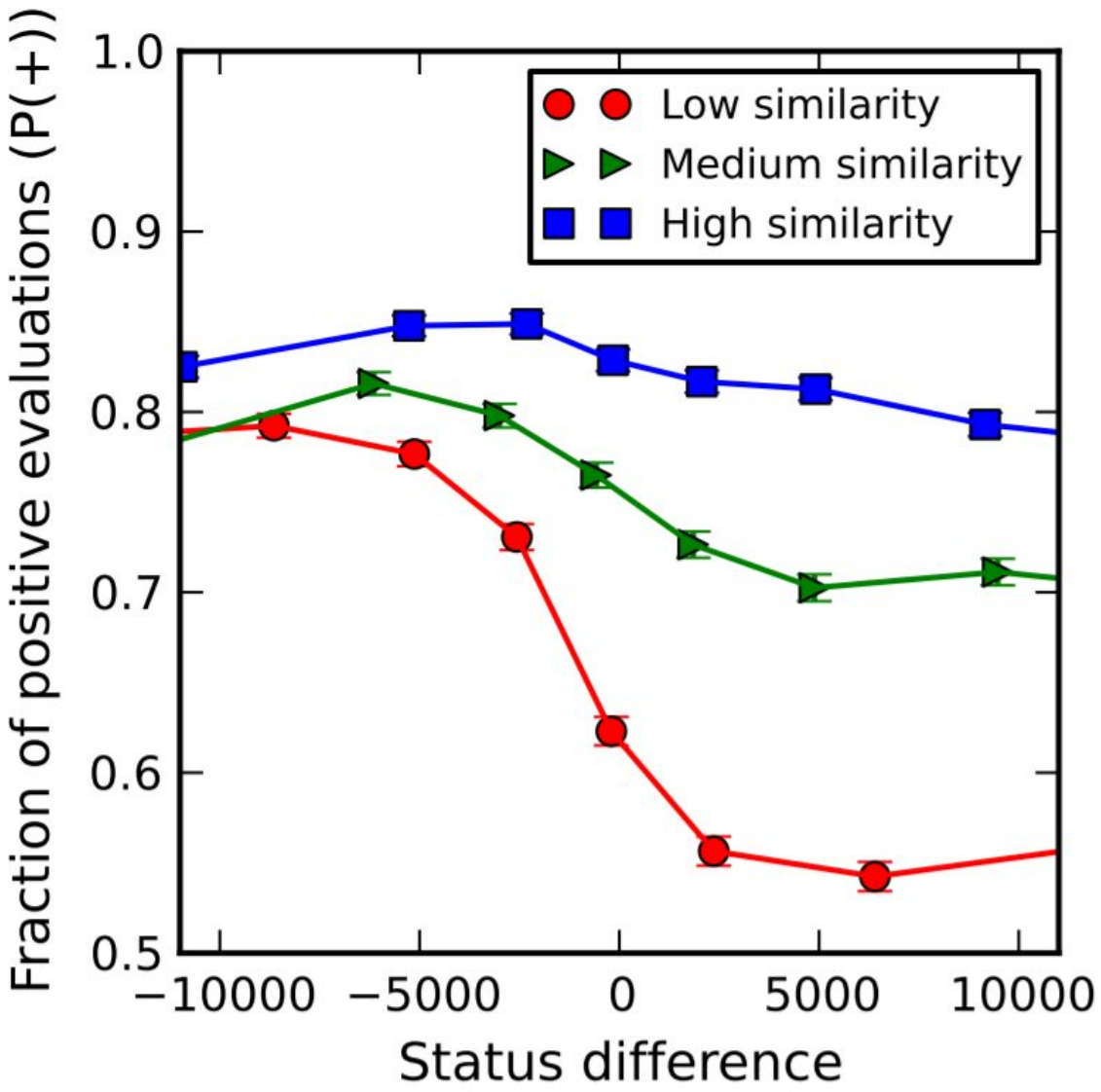
- People are more supportive to other people in their domain of knowledge/area
  - "The more similar you are, the more I like you"

- People know the domain, hence know the weak point and are more harsh
  - "The more similar you are, the better I can understand your weaknesses"

Effect of Similarity



Similarity and Status



Summary so far

- Online Social Networks and Social Media websites support (sometimes implicitly) user evaluations (e.g. Wikipedia has transparent mechanism of elections)

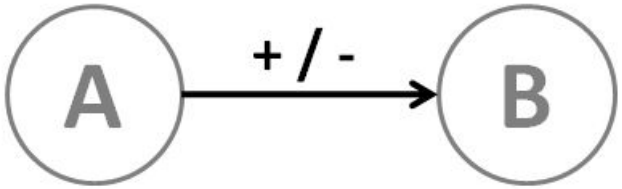
- Two important characteristics:
  - Status: importance of relative assessments
  - Similarity: importance of prior interactions

More info: https://cs.stanford.edu/people/jure/talks/evals-recsys-sep12.pdf

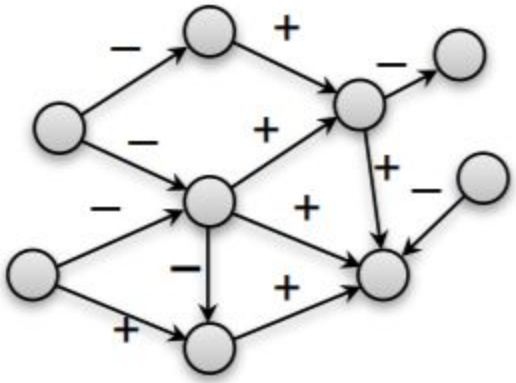
Evaluation – with context

Two ways to look on it:

- Single evaluation (without network context)

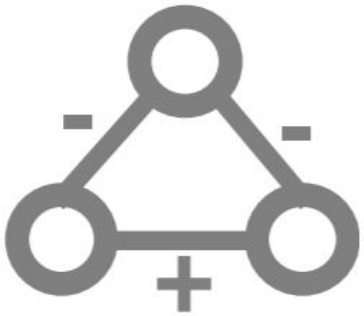


- Evaluations in the context of the network



Networks with Signed Edges

- Also called: “Signed Network”
- Basic unit of investigation: **Signed triangles**
- Can be undirected or directed:



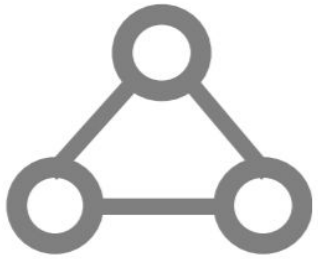
Signed Networks

- Network with **positive** or **negative** relationships
- Consider a complete signed undirected graph
  - **Positive** edges:
    - Friendship, positive sentiment, …
  - **Negative** edges:
    - Enemy, negative sentiment
- Let's focus on three connected nodes A, B, C

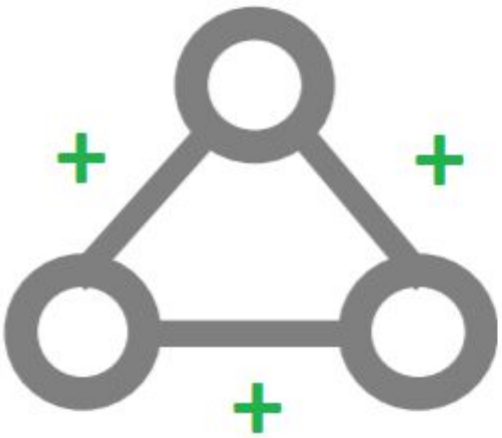
Theory of Structural Balance

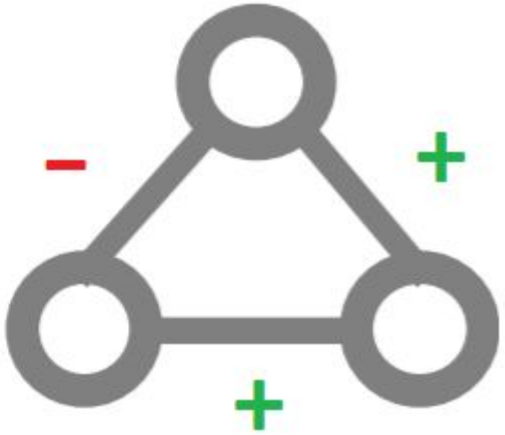
- Intuition (theory by Fritz Heider 1946):
  - **Friend** of a **friend** is a **friend**
  - **Enemy** of an **enemy** is a **friend**
  - **Enemy** of a **friend** is an **enemy**

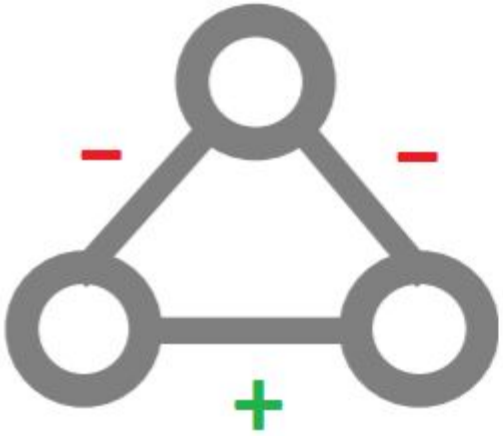
- Let's have a look on a triangle in a graph

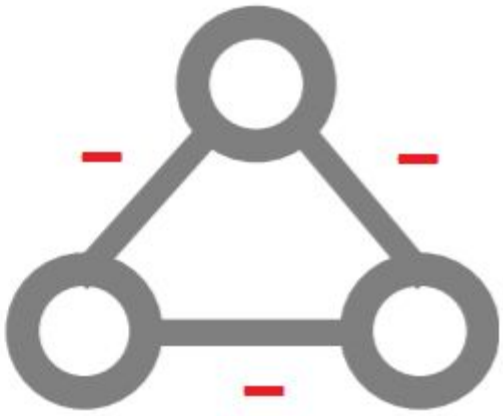


Balanced/Unbalanced Triangles

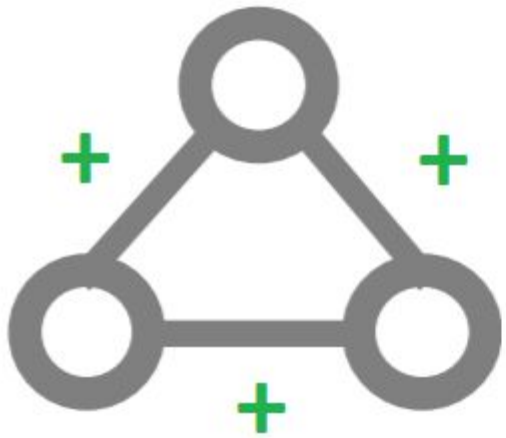




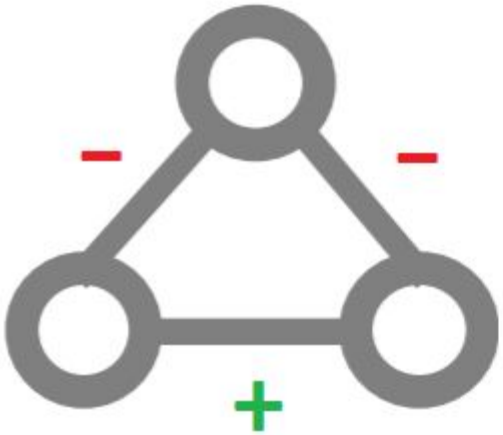


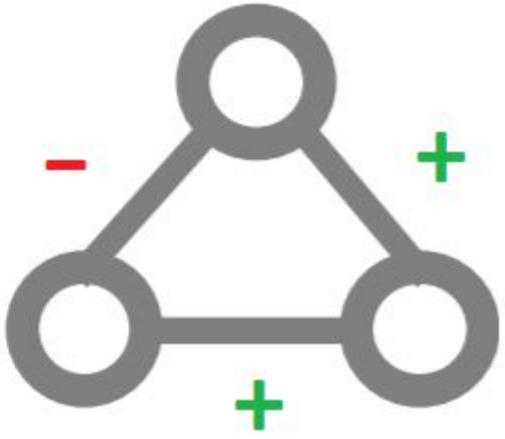


Balanced/Unbalanced Triangles

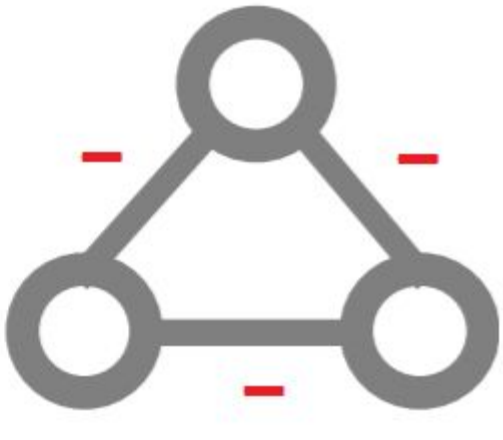


Balanced



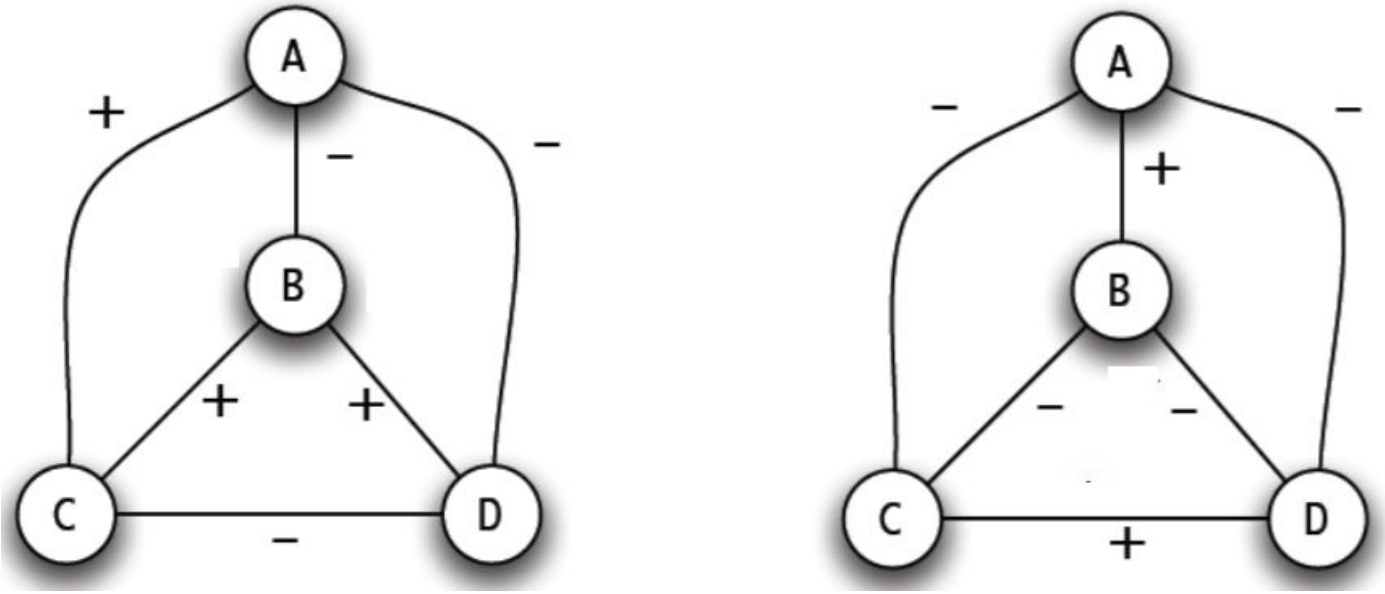


Unbalanced



Balanced/Unbalanced Network

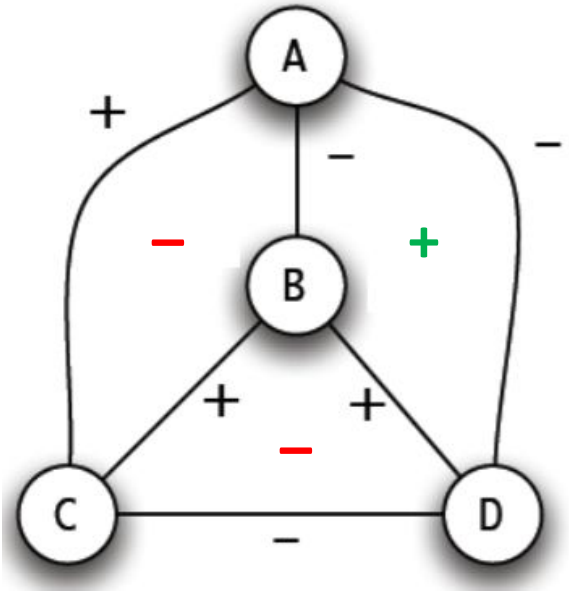
- Network is balanced if every triangle in the network is balanced.



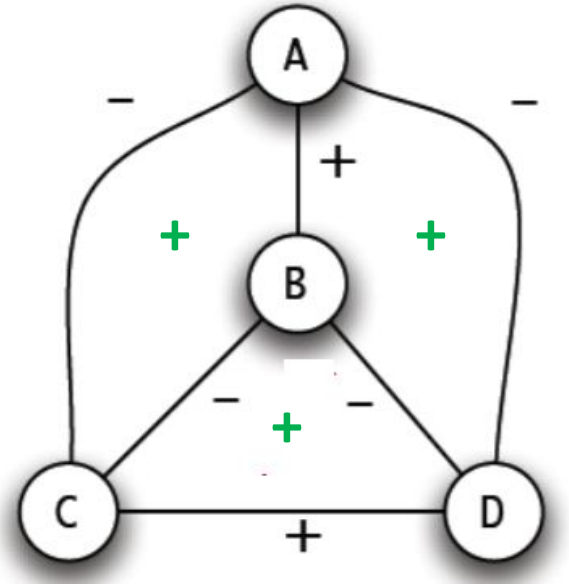
- Balanced triangle – 1 or 3 “+” edges

Balanced/Unbalanced Network

- Network is balanced if every triangle in the network is balanced.



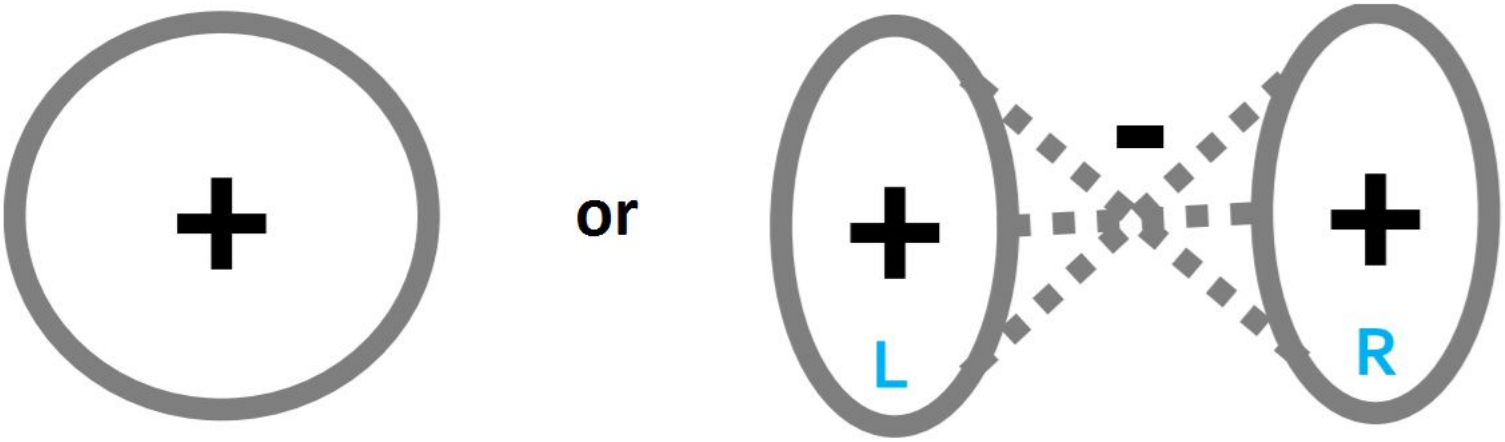
Unbalanced



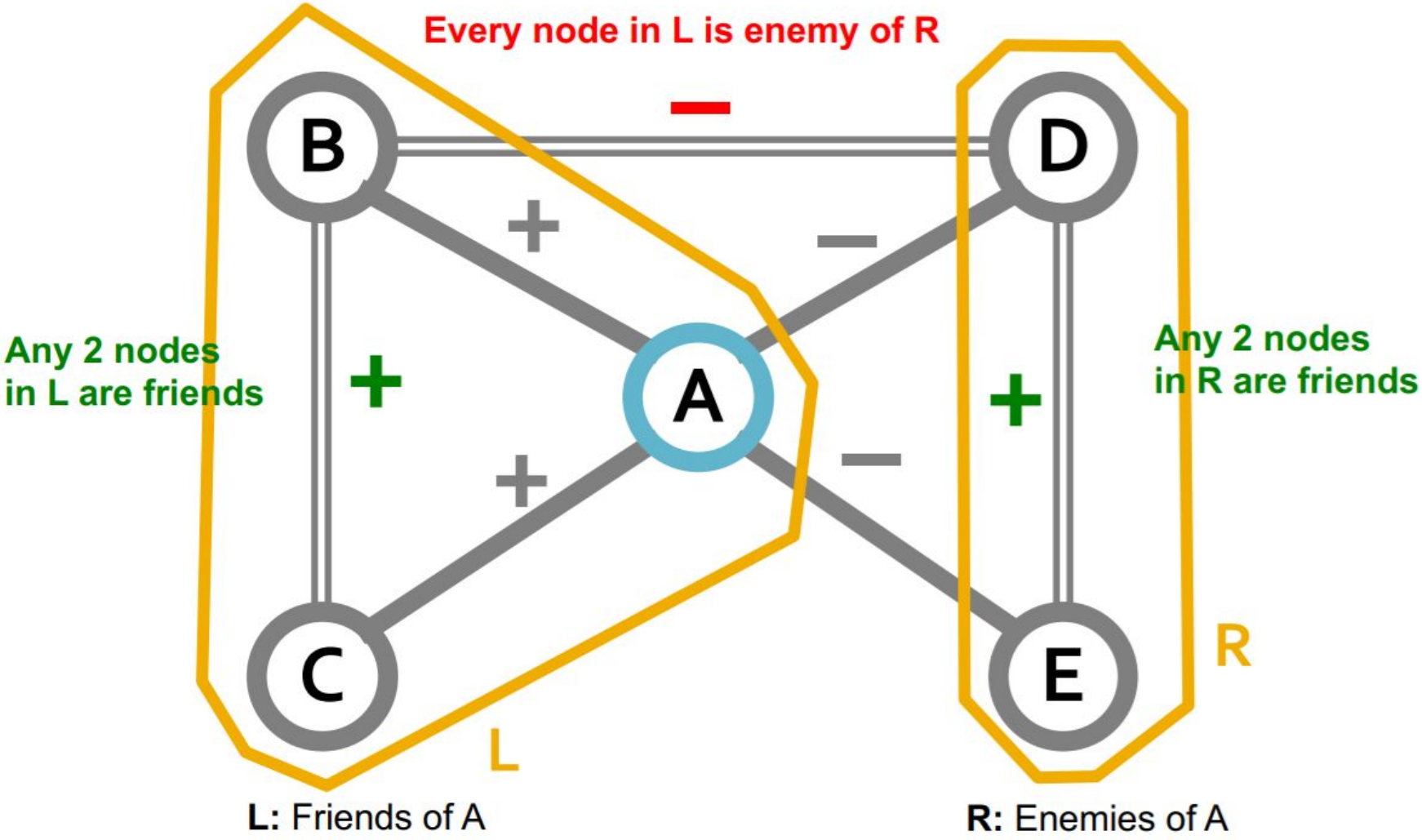
Balanced

Balance and Coalitions

- If the network is balanced, then either:
  - All edges are positive, or
  - We can split the network into two parts (L and R),
    - All edges inside R are **positive**
    - All edges inside L are **positive**
    - All edges between R and L are **negative**



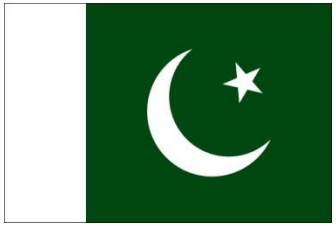
Analysis of Balance: Coalitions



Example: International relations

Independence of Bangladesh from Pakistan in 1971

USA supported Pakistan. Why?



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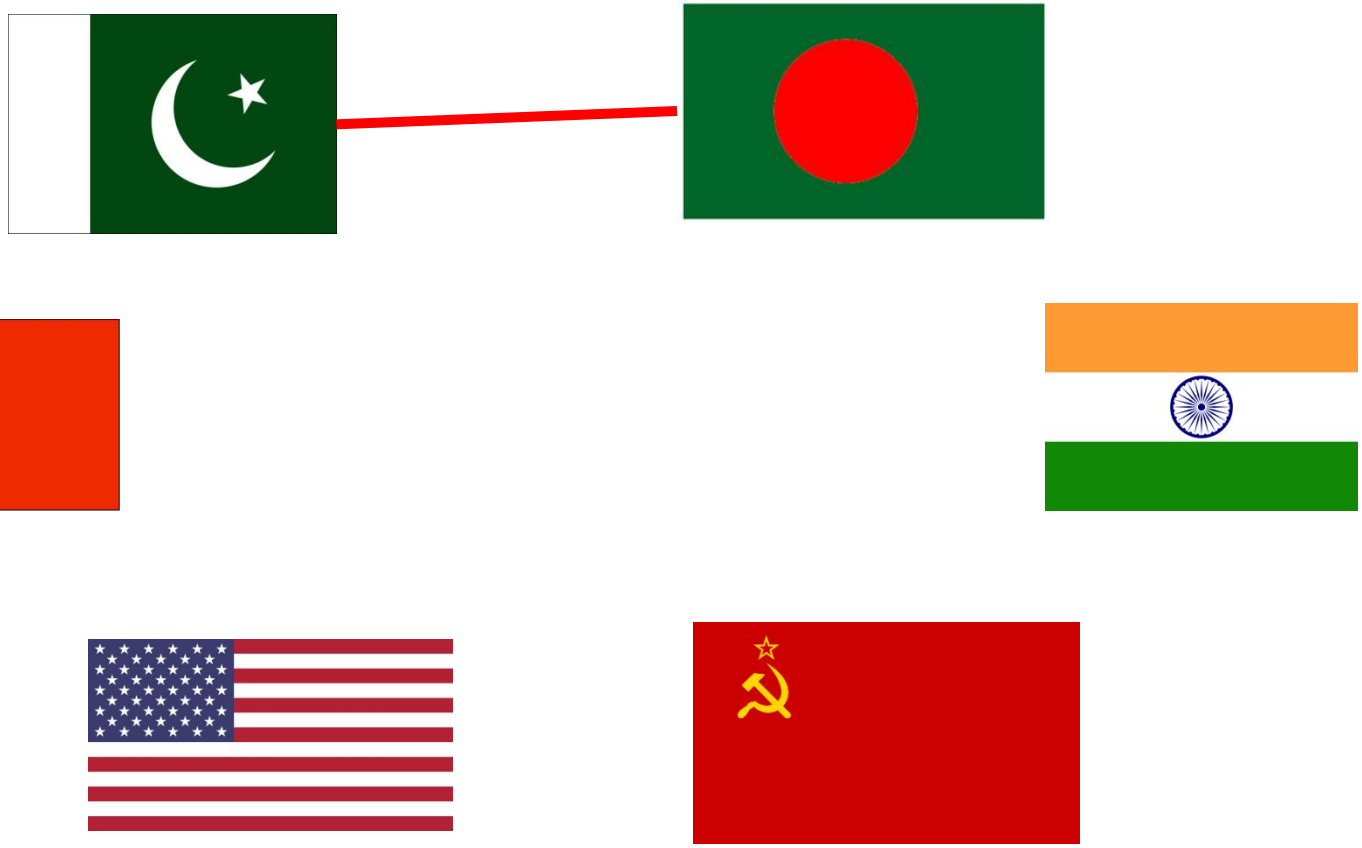
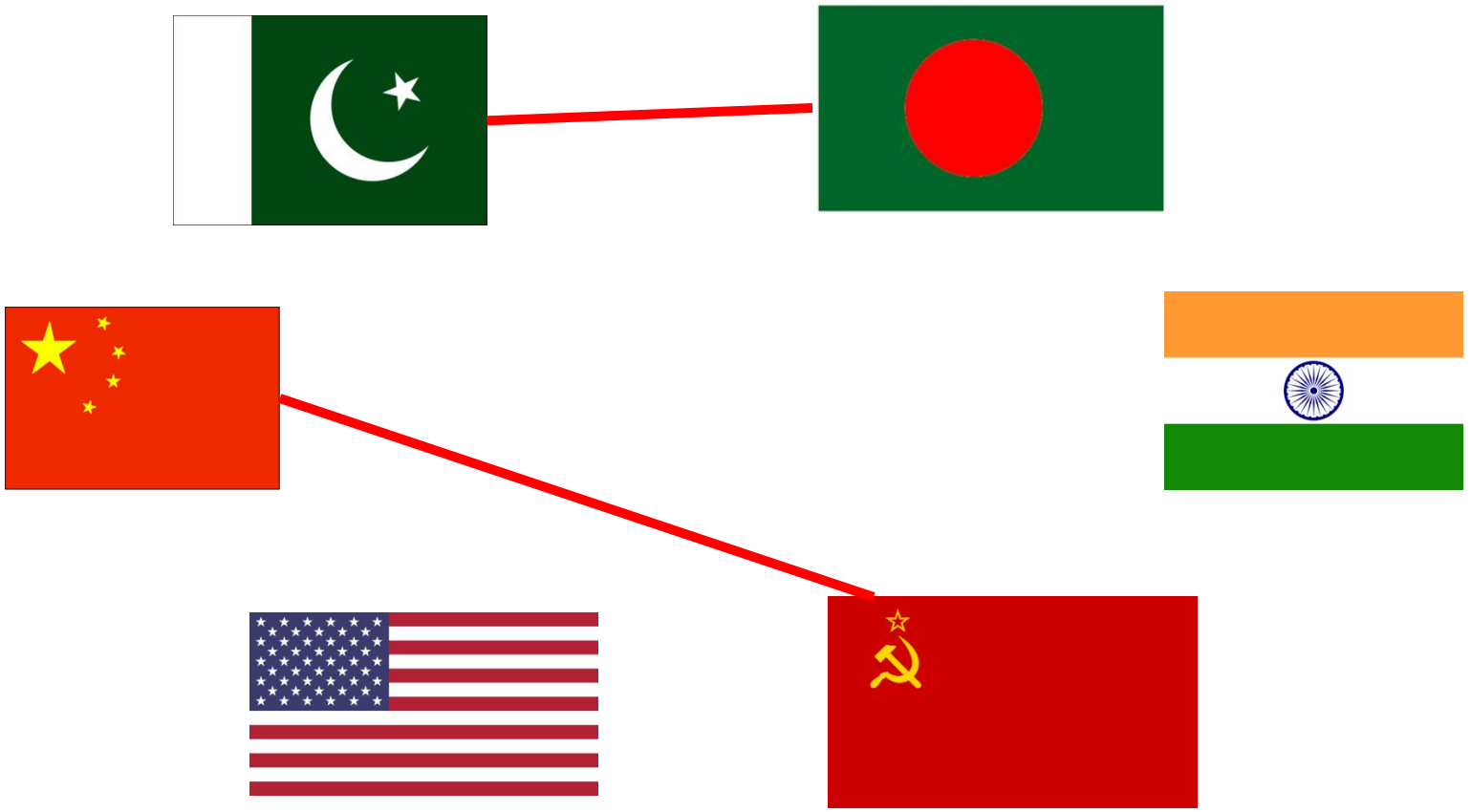
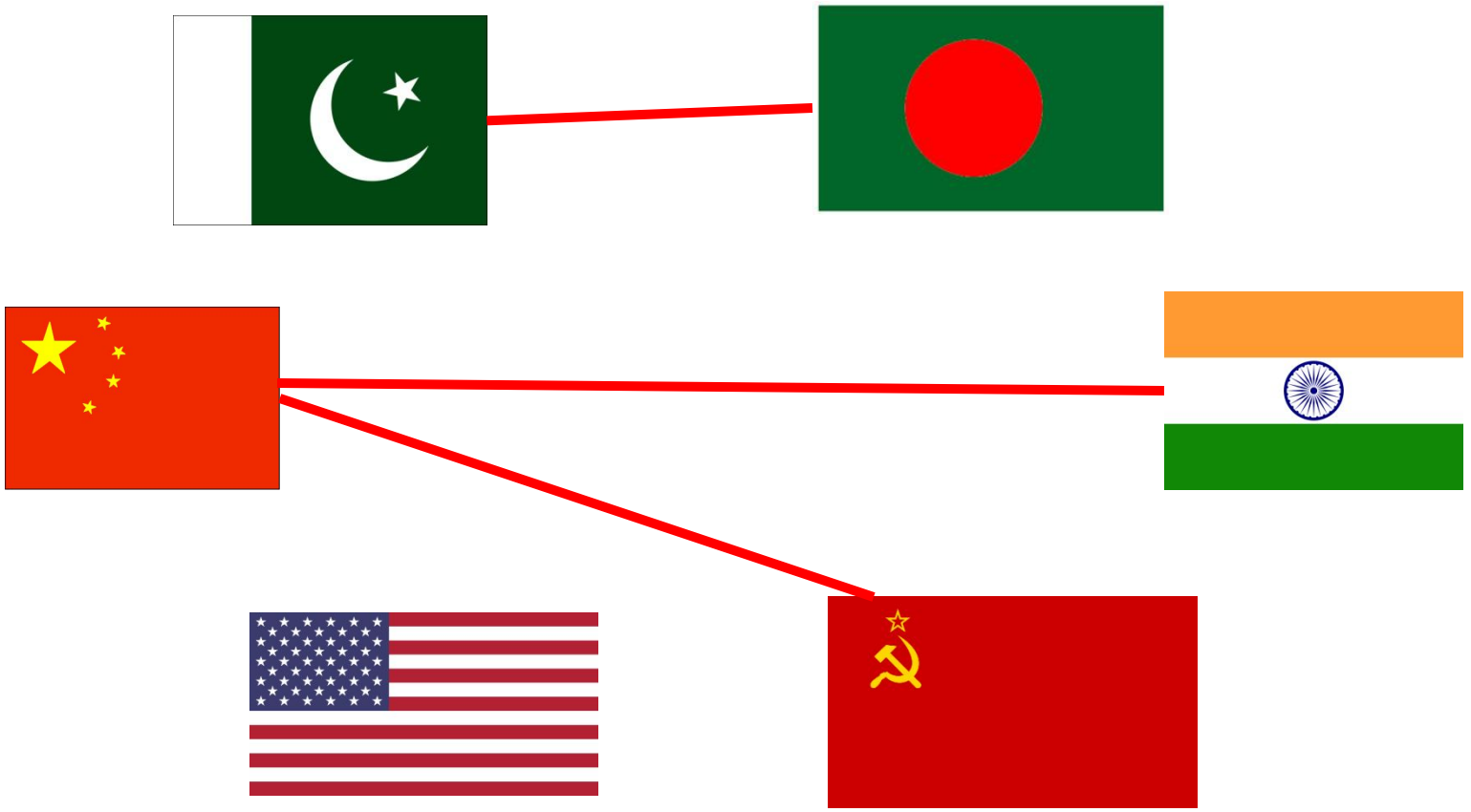


Example: International relations



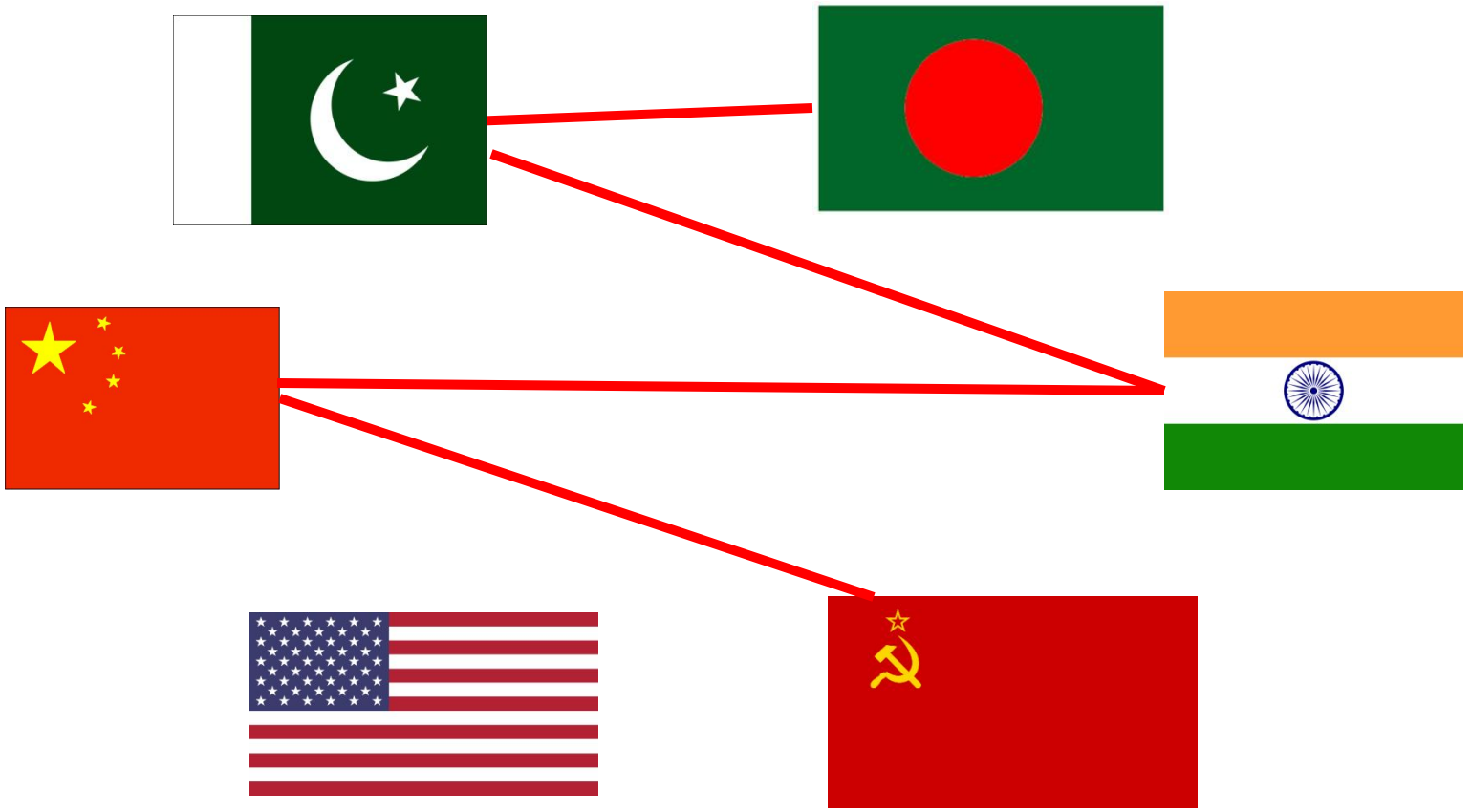
USSR is an **enemy** of China

Example: International relations



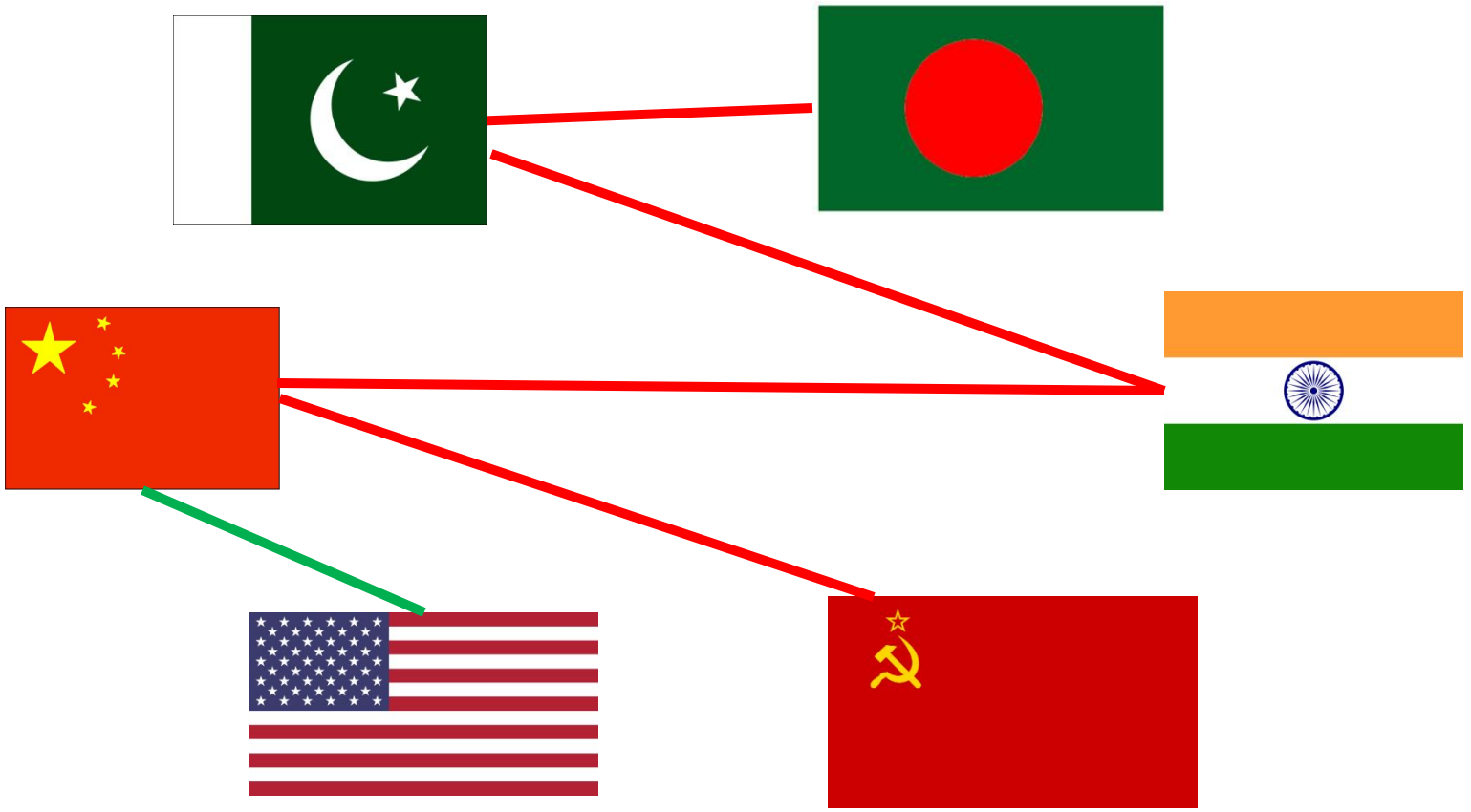
China is an **enemy** of India

Example: International relations



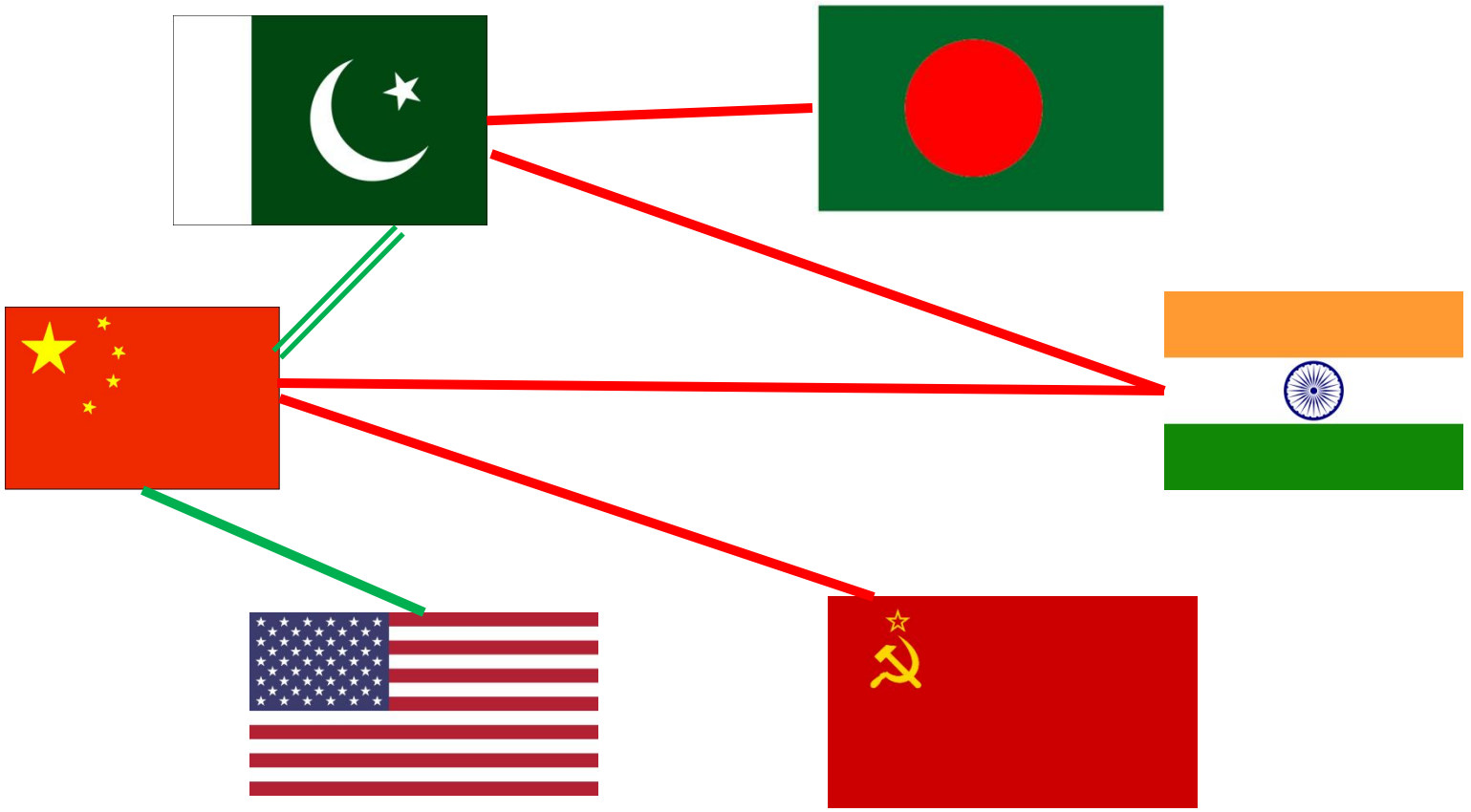
India is an **enemy** of Pakistan

Example: International relations



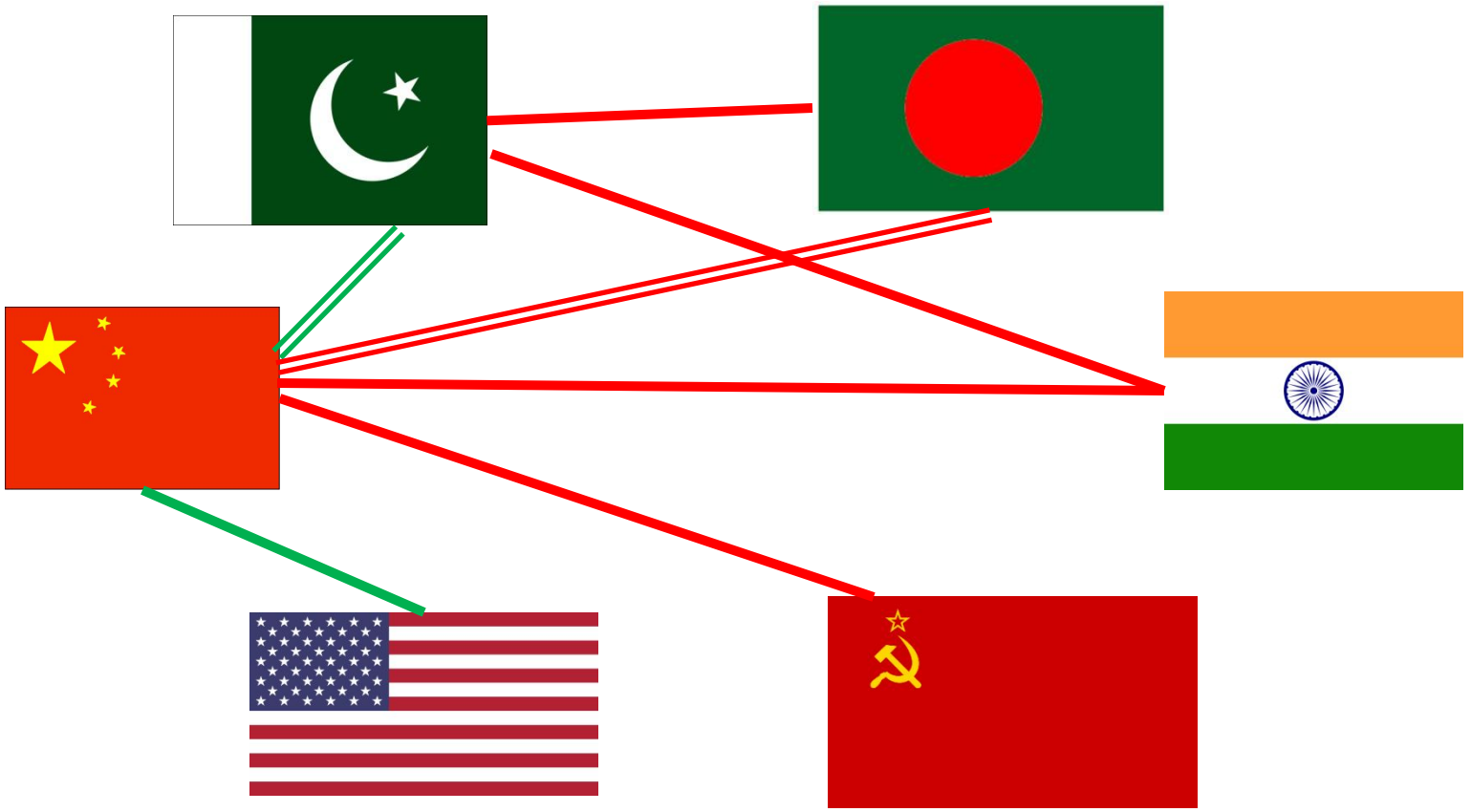
USA is a **friend** of China

Example: International relations



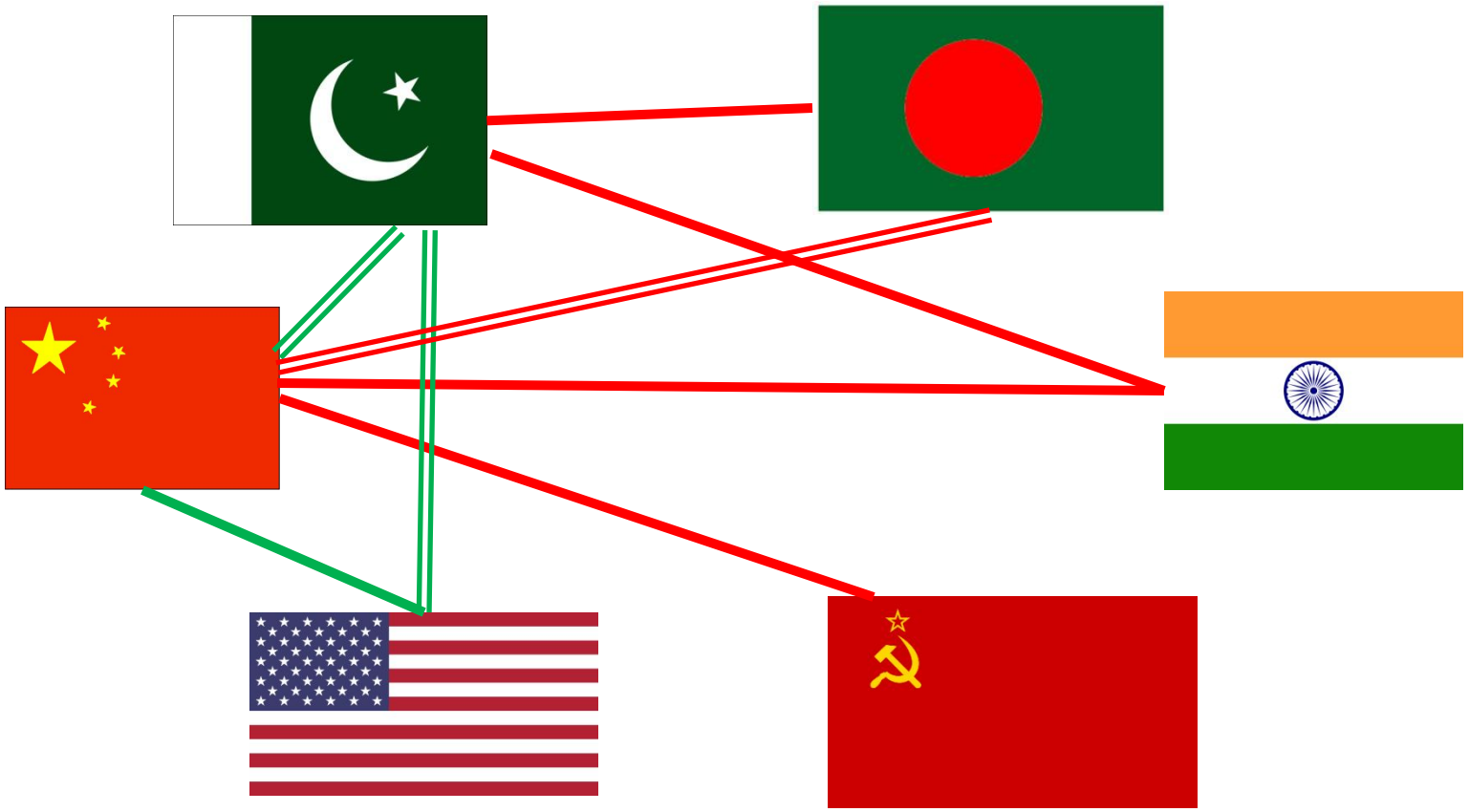
Derived: China is a **friend** of Pakistan

Example: International relations



Derived: China is **vetoed** of Bangaladesh

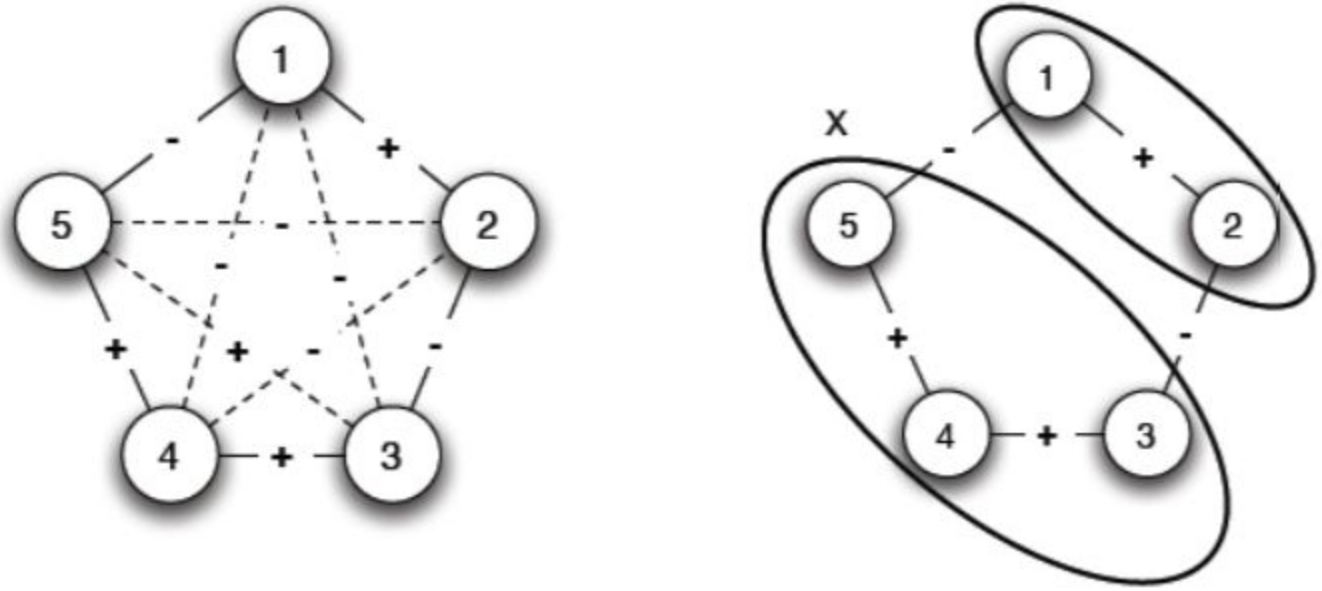
Example: International relations



Derived: USA **supported** Pakistan

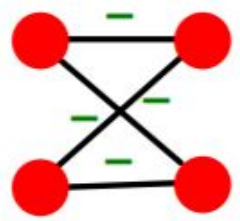
Balance in General Network

- The (general) network is balanced if:
  - We can fill all missing edges to achieve balance
  - We can divide the network into two coalitions



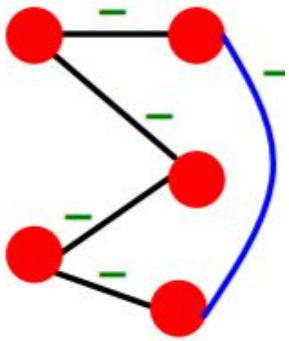
Check if Network is Balanced

- Graph is balanced if and only if it contains no cycle with an odd number of negative edges



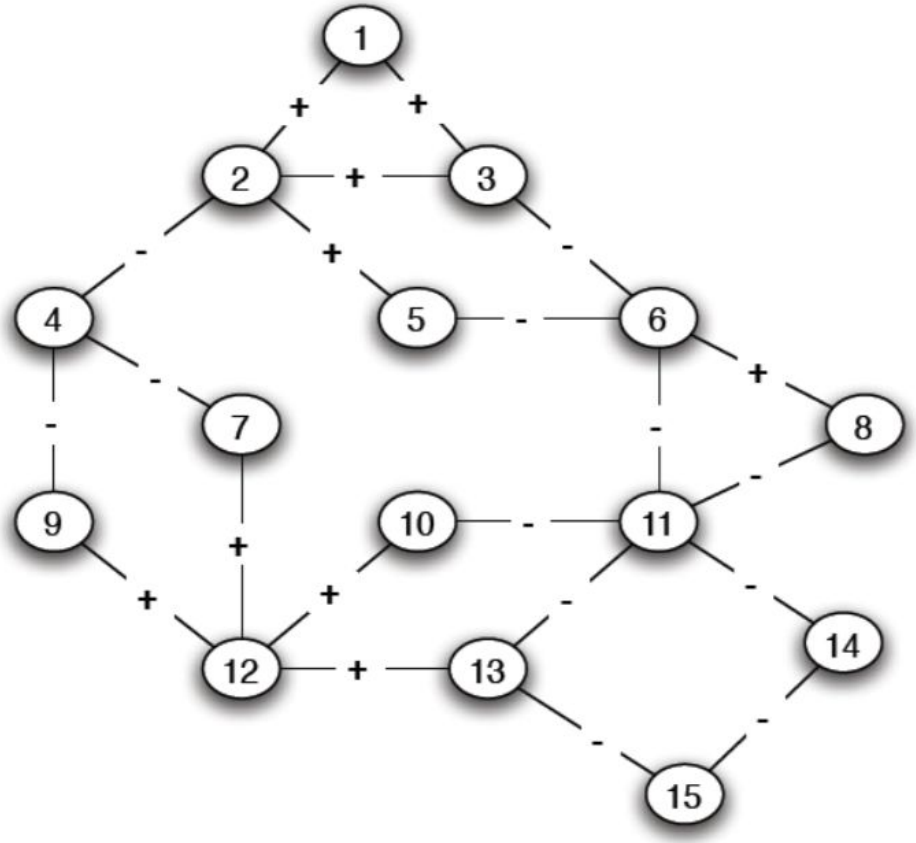
Even length cycle

- Find connected components on +edges
  - If we find a component of nodes on +edges that contains a –edge
    - Unbalanced
- For each component create a super-node
- Connect components A and B if there is a negative edge between the members
- Assign super-nodes to sides using BFS

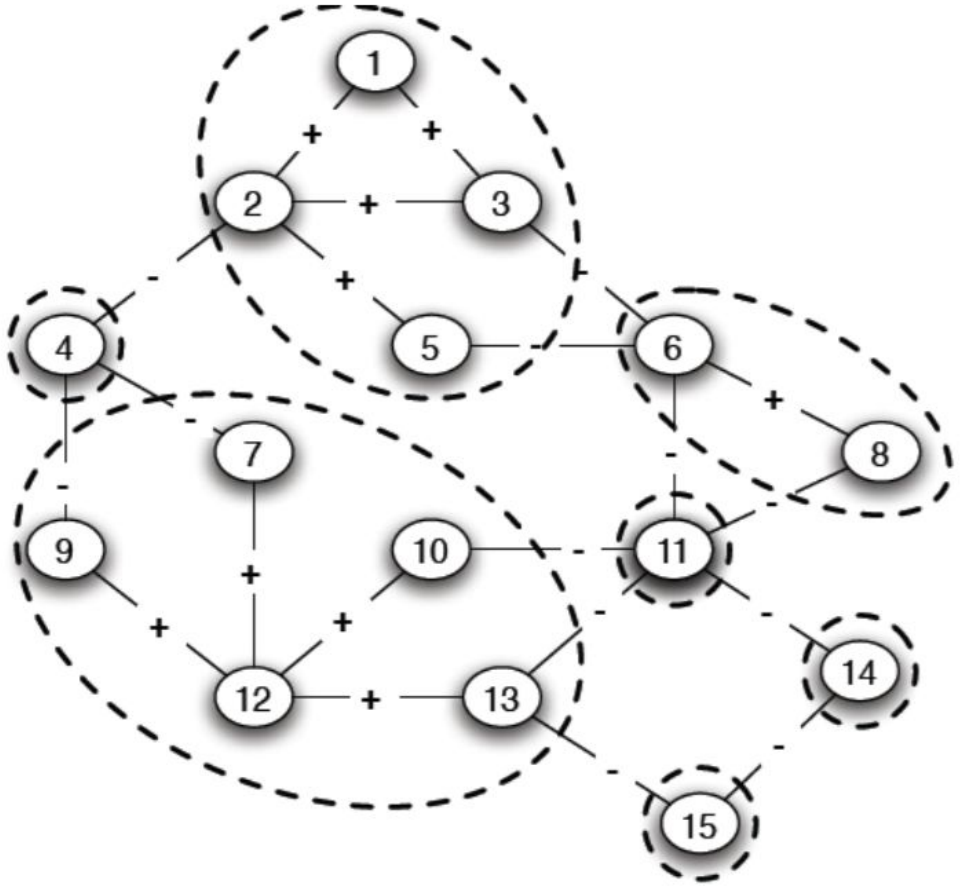


Odd length cycle

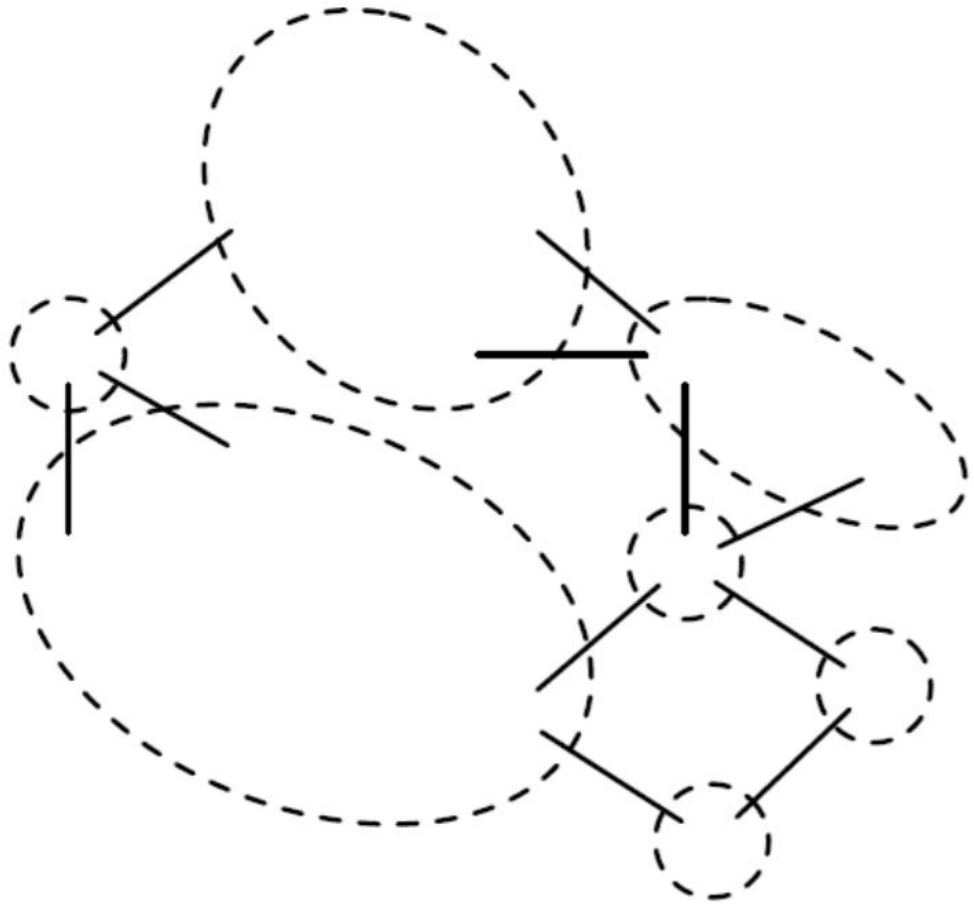
Check if Network is Balanced



Check if Network is Balanced

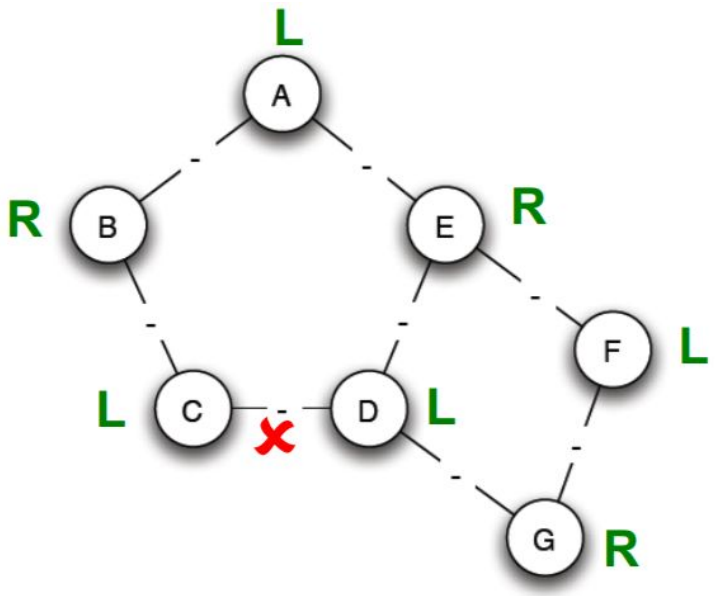


Check if Network is Balanced



Check if Network is Balanced

- Using BFS – assign each node to a side
- Graph is **unbalanced** if any two connected super-nodes are assigned the same side



Thank you!

Questions?