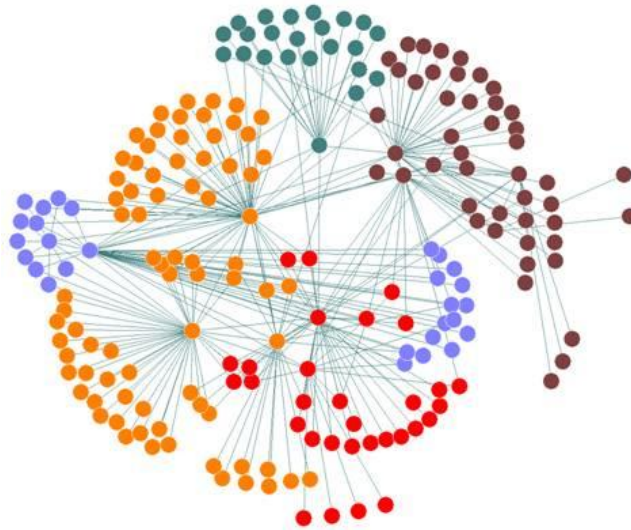




Algorithms and Applications in Social Networks



2024/2025, Semester A
Slava Novgorodov

Lesson #1

- Administrative questions
- Course overview
- Introduction to Social Networks
- Basic definitions
- Network properties

Administrative questions

- Course format:
 - Lecture (2h) + Recitation (1h) every week (by Slava)
 - 3 Homework tasks during the semester
 - Submission in pairs
 - Theoretical + Practical (Python) questions
 - Final exam (format will be discussed later)
 - Final grade = 85% Exam + 15% HW

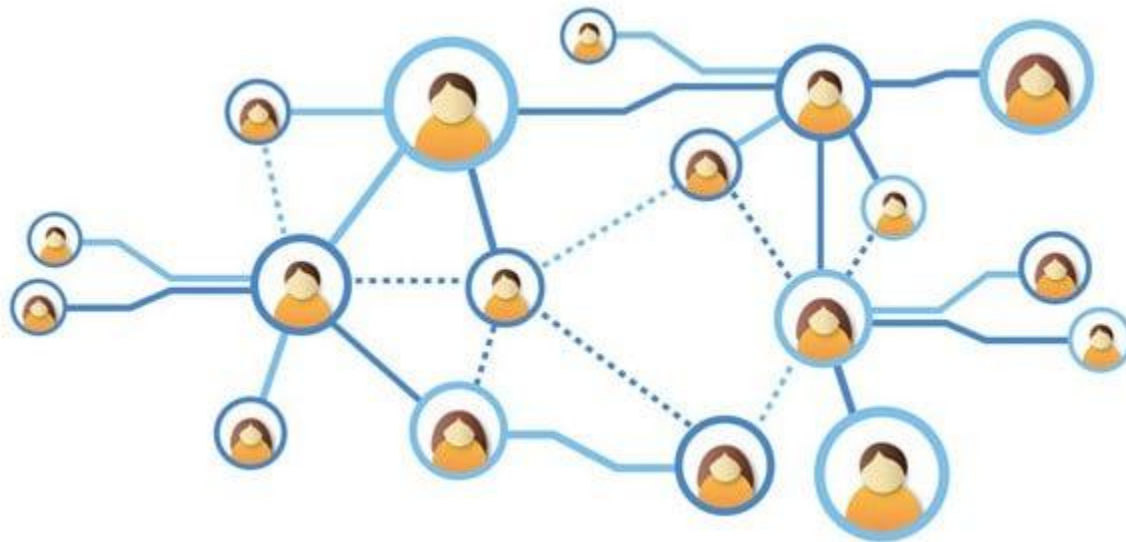
 - Office hours – Sunday (schedule in advance)
 - Course website:
<https://slavanov.com/teaching/sn2425a/>
 - Email: slavanov@post.tau.ac.il (**not** mail.tau.ac.il !)

Related material

- Books:
 - **Newman** “Networks: An Introduction”
 - **Jackson** “Social and Economic Networks”
 - **Easley & Kleinberg** “Networks, Crowds, and Markets: Reasoning About a Highly Connected World”
<http://cs.cornell.edu/home/kleinber/networks-book/>
 - **Wasserman & Faust** “Social Network Analysis. Methods and Applications.”
- Related courses:
 - CS224W (Stanford) – Analysis of Networks
<https://web.stanford.edu/class/cs224w/>
 - Social and Economics networks (online course)
<https://www.youtube.com/channel/UCCnG8fKY45aH73ahmGK2xcg>
 - High School of Economics – Social Networks
<http://leonidzhukov.net/hse/2014/socialnetworks/>

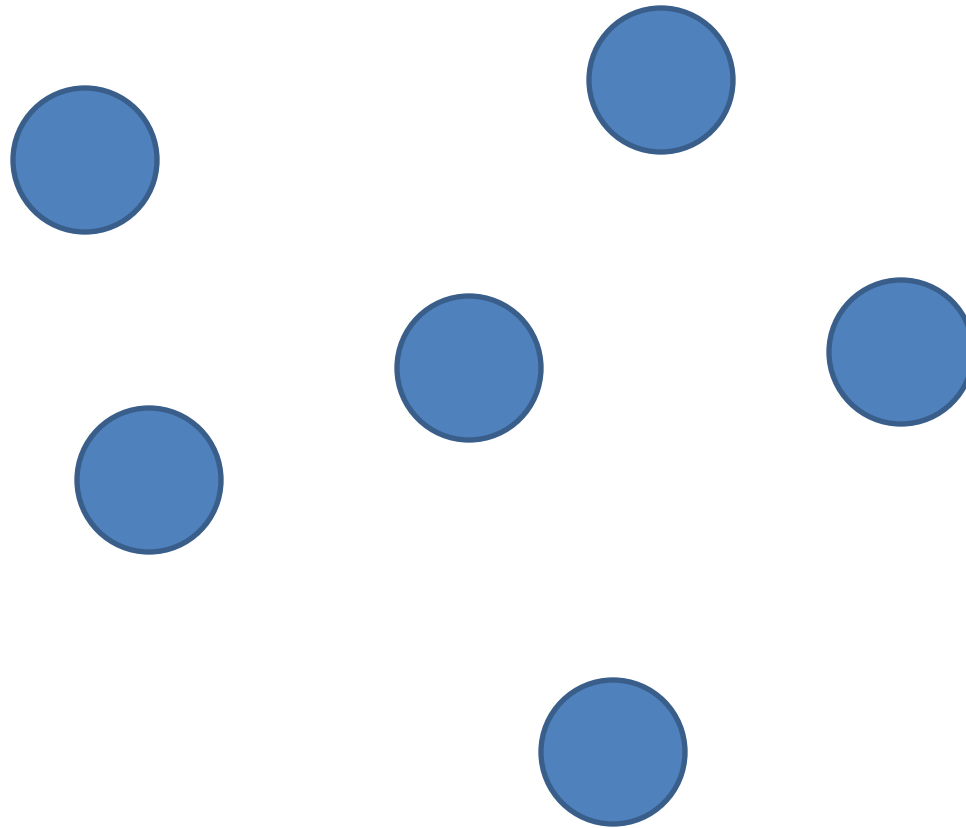
Social Networks

- **Social Network** - a structure of social actors (individuals or organizations) and social interactions between the actors



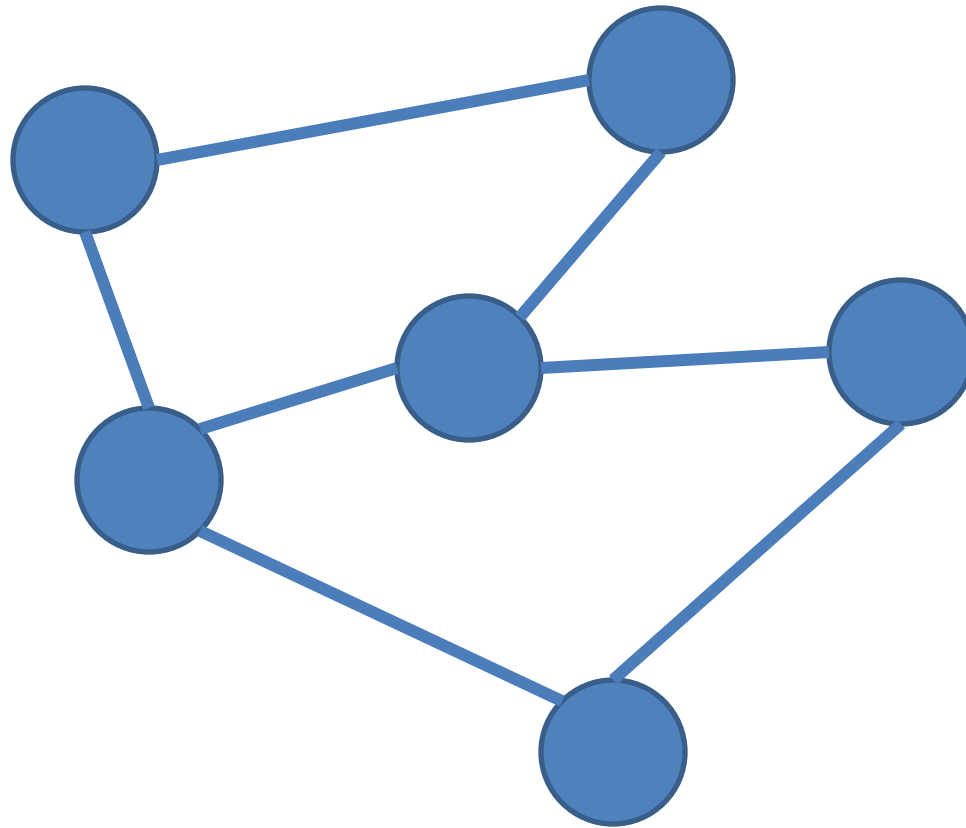
Social Networks

Social Networks



Social actors

Social Networks



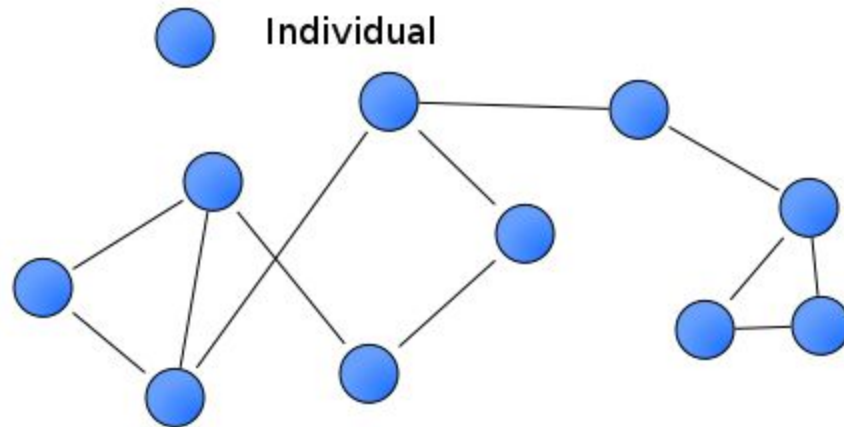
Social actors and **interactions**

Social Networks

- **Interdisciplinary** field, studied in:
 - Sociology
 - Social psychology
 - Economics
 - Statistics
 - Mathematics (Graph Theory)
 - Computer Science (this course)

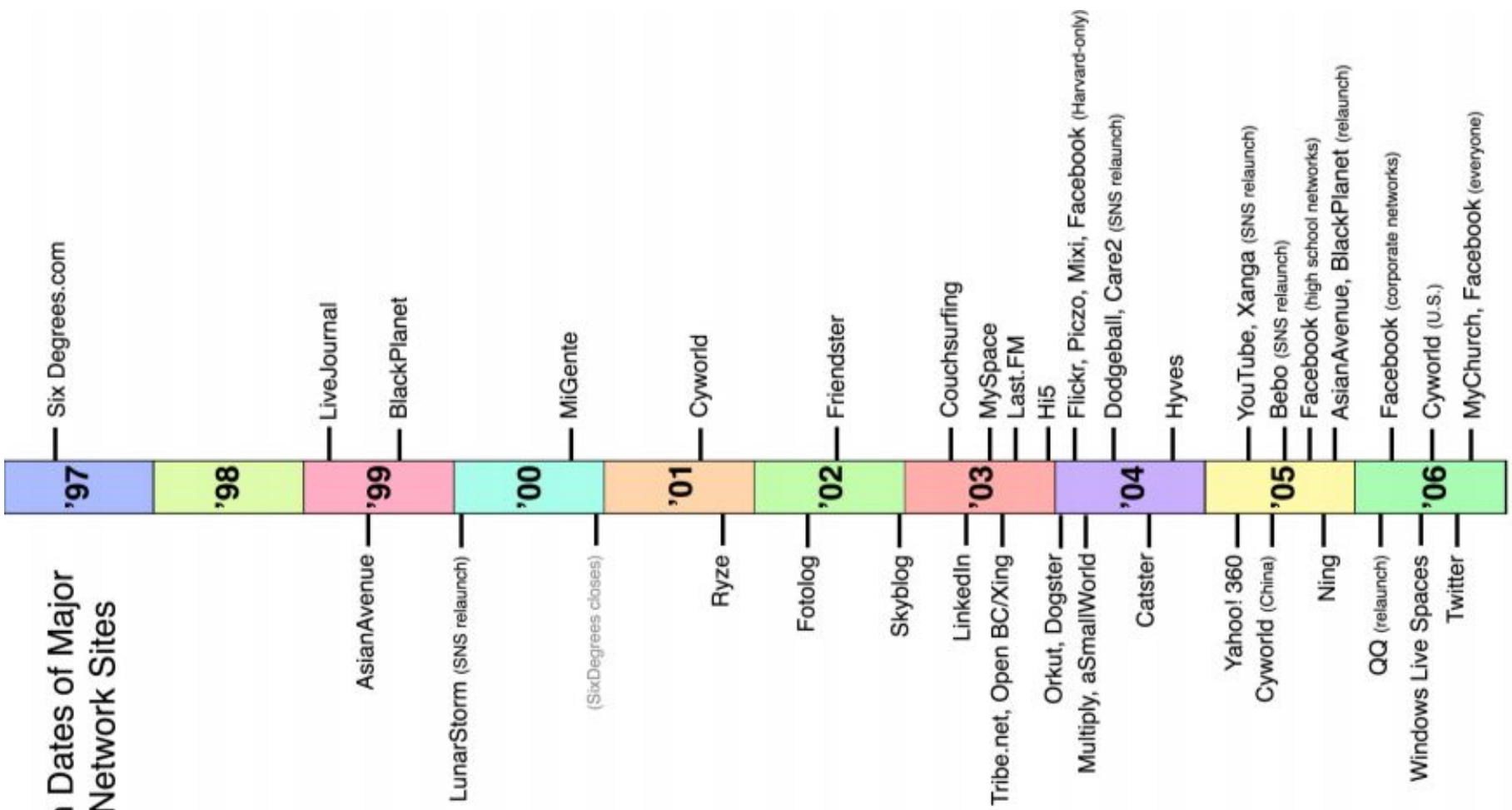
Social Networks

- The research around Social Networks started at the beginning of 1930s (first sociograms)



- Mathematical formulation – 1950s
- 1980s and later – growth in number of social network research and researchers
- Late 1990s until now – **online** social networks

Launch Dates of Major Social Network Sites



Research clusters

- Communications
- Complex networks
- Criminal networks
- Spread of innovations
- Demography
- Health care
- Language and linguistics
- Social media
- ...

What can be presented as SN?

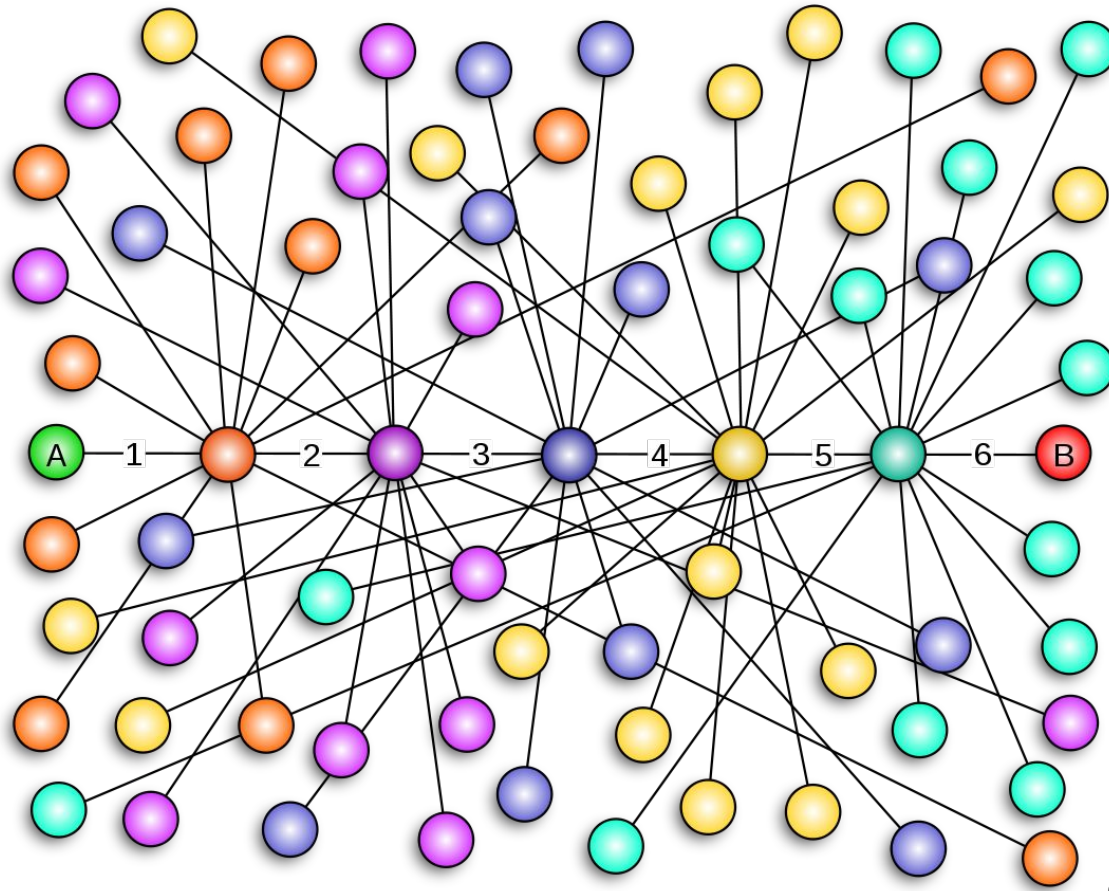
- Friendship and other social relationships
- Corporative structures (internal/external)
- Trade relationships (individuals/companies)
- Political alliances
- Sharing of information
- Criminal organizations structures
- ...

Three aspects

- Theory
 - Network formation, dynamics...
 - Influence detection
 - Communities
- Experimental studies
 - Observe patterns
 - Test theories
- Methodology
 - How to analyze networks?

Applications in Social Networks

6 degrees of separation



dw 2010

6 degrees of separation

The Small World experiment:

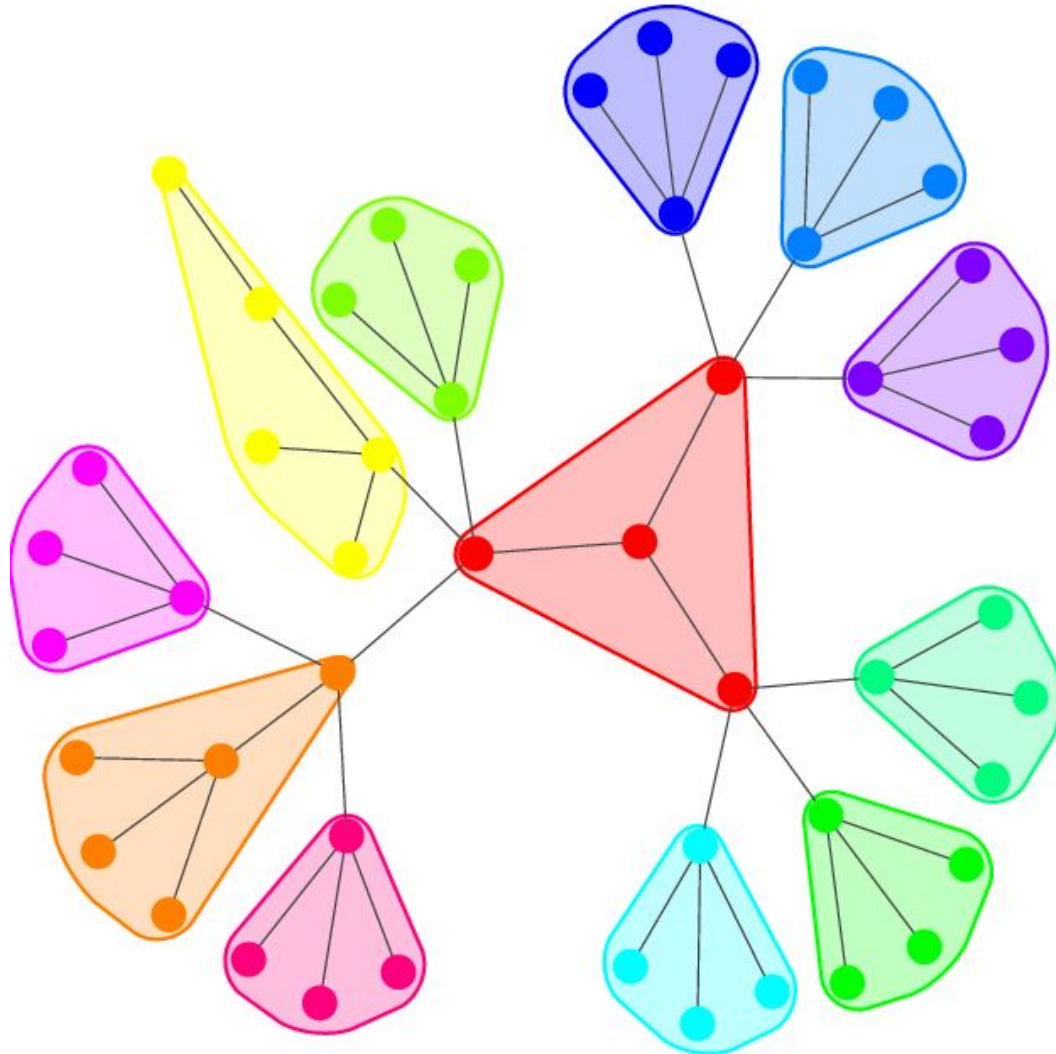
Model the population as a social network and attempt to find the average path length between any two nodes.

1. Select individuals in two far (socially and geographically) points – Omaha, Nebraska and Boston, Massachusetts
2. The individual in Omaha received a letter he/she needs to pass to an individual in Boston. If they know each other, great. Otherwise, the letter should be sent to a friend who may know the destination individual.

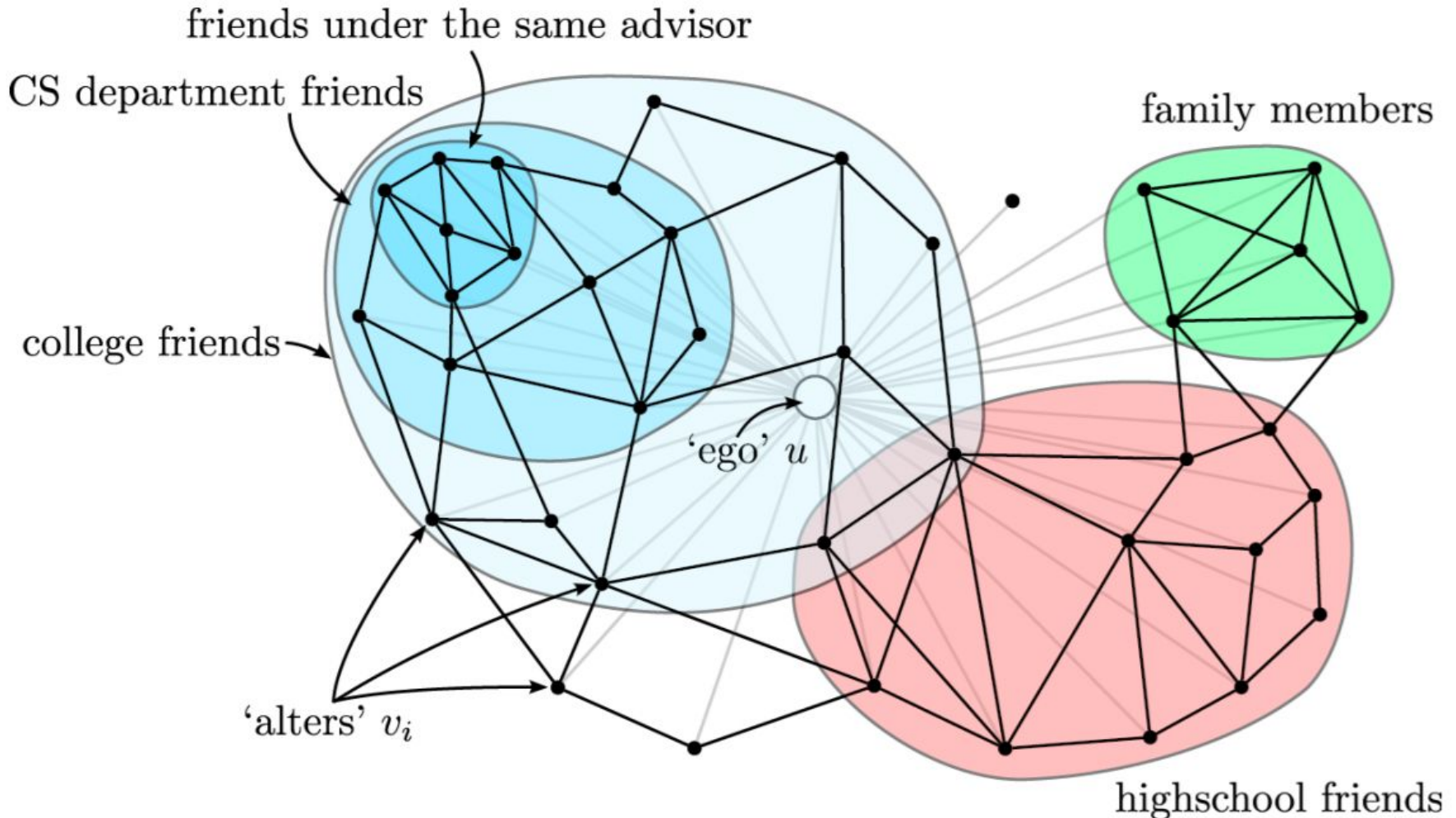
Results: 64 letters reached the target within 5.5 hops on average

Facebook case: Around 4 degrees of separation (<https://arxiv.org/abs/1111.4570>)

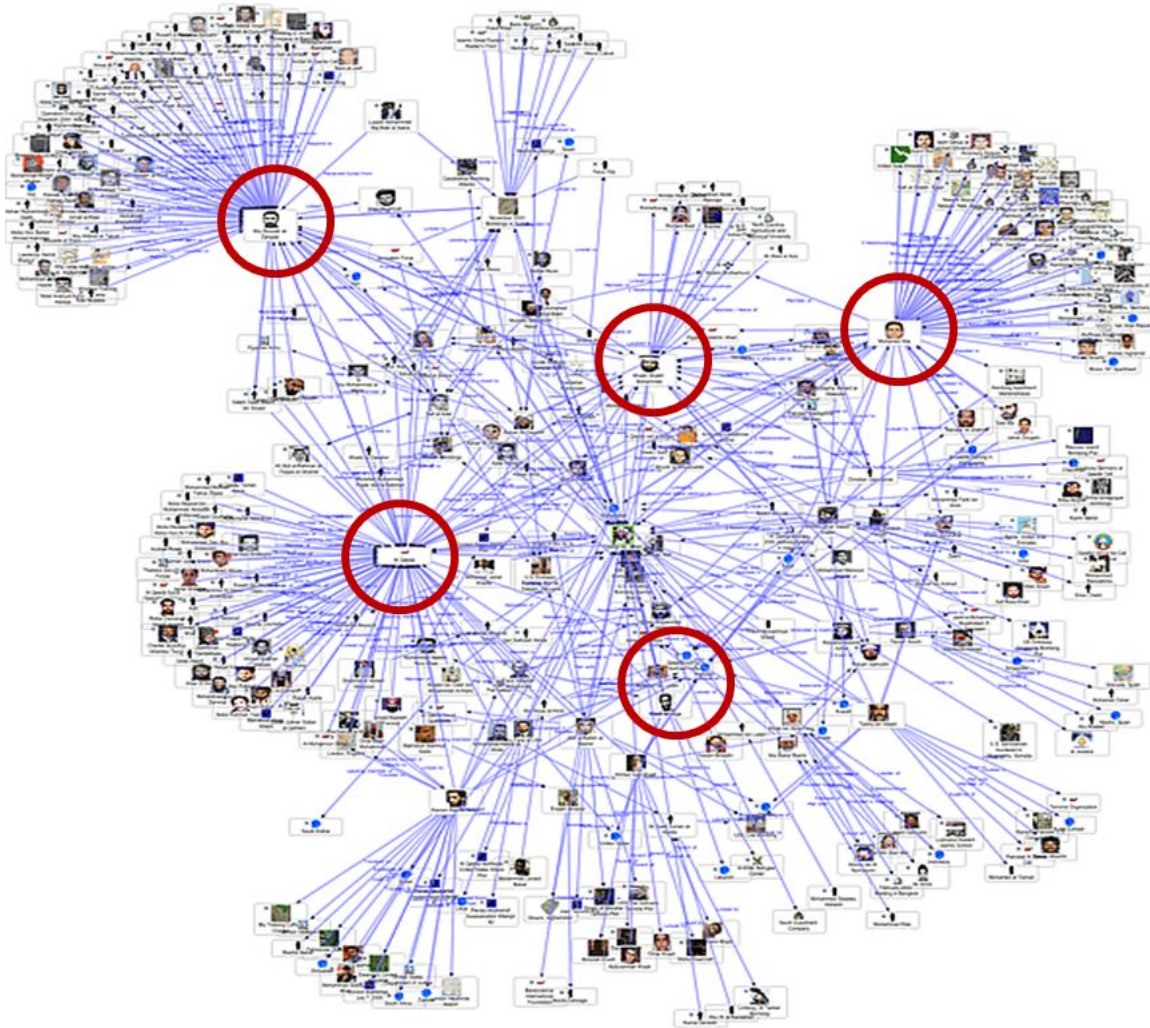
Community detection



Community detection



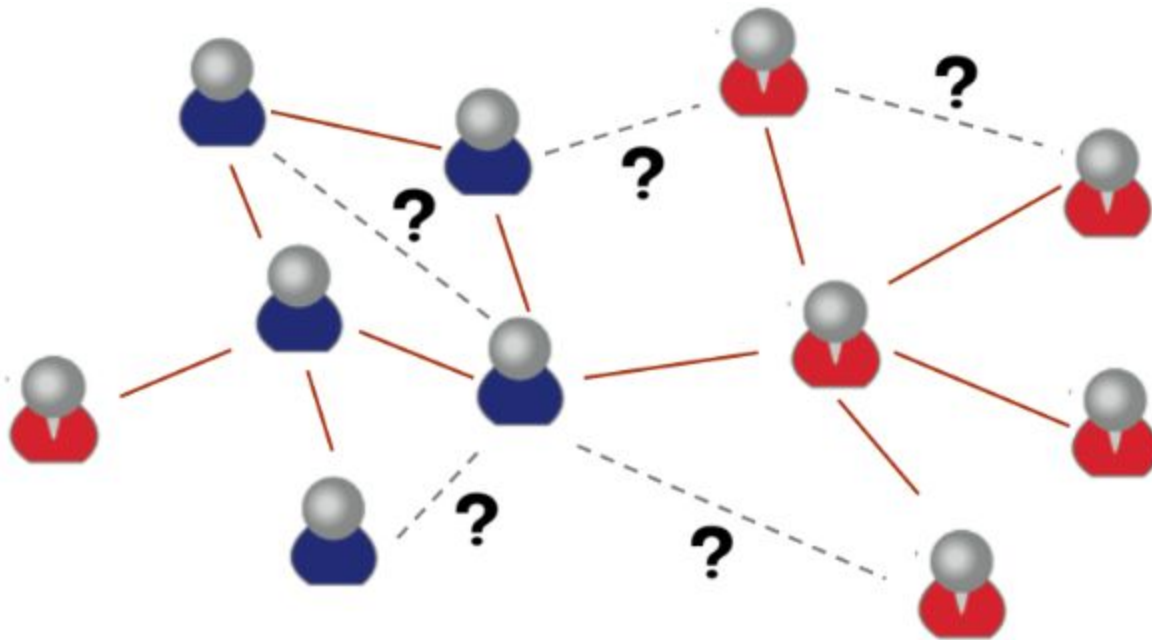
Influence Maximization



Find K individuals in the social network that maximize the influence

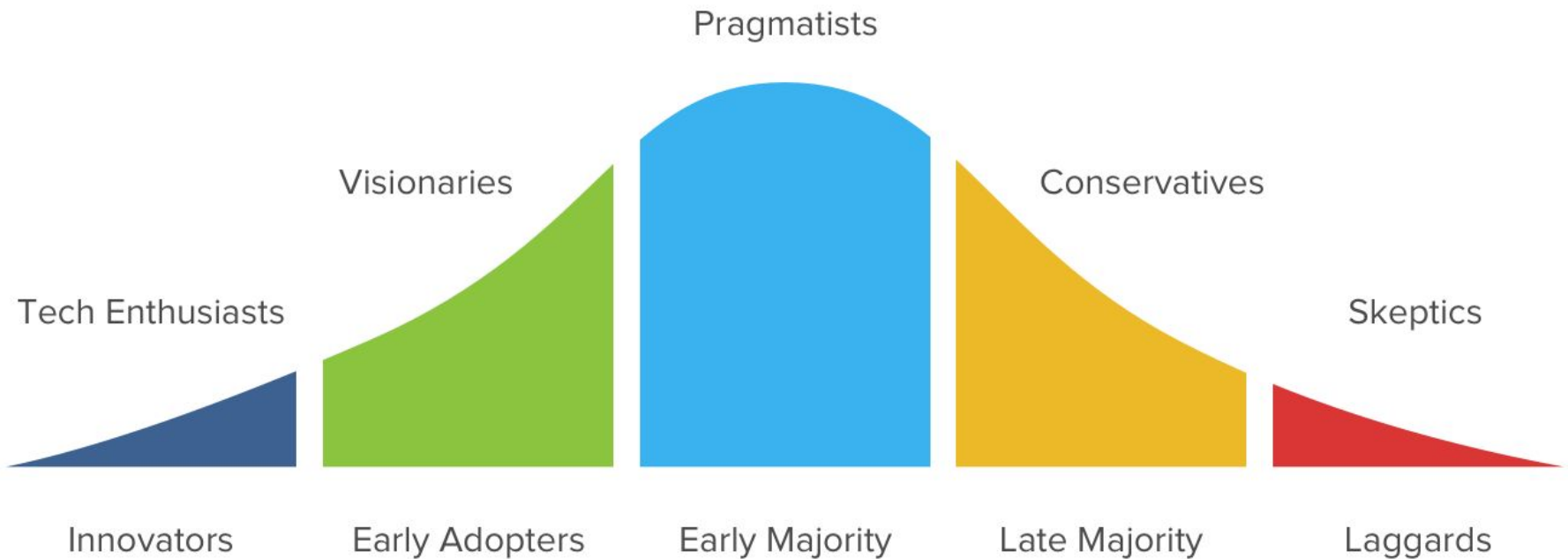
Link prediction

- “Suggested friends” feature

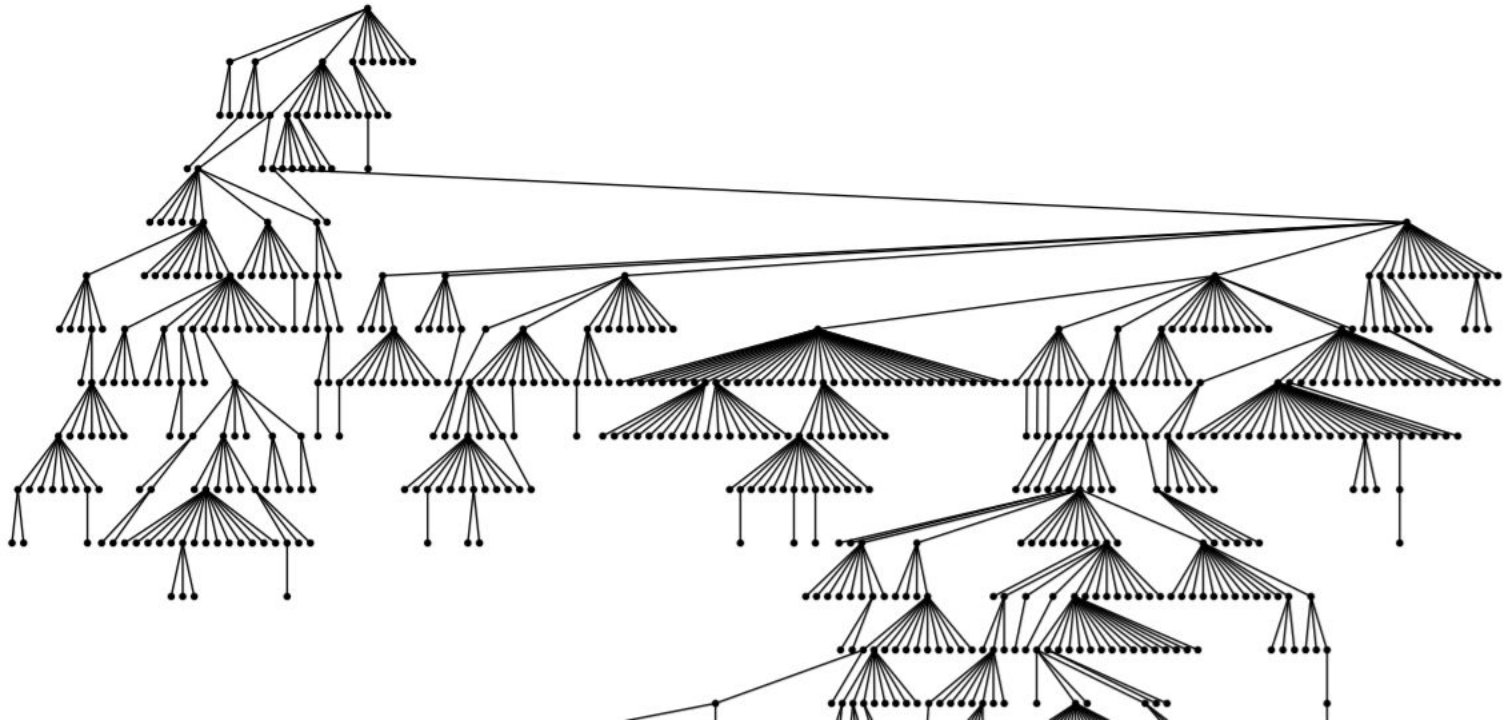


Product adoption

Product Adoption Curve



Product adoption



60% to 90% of LinkedIn users registered from friends invitation
(Anderson, Huttenlocher, Kleinberg, Leskovec, Tiwari, WWW'15)

Misinformation detection



Analyzing the content of the information and also the **source** and **pattern of spread**

Fake accounts detection



Detecting fake accounts using behavioral analysis

And more...

- Fraud financial activities
- Spread of diseases
- Employee and companies success
- ...

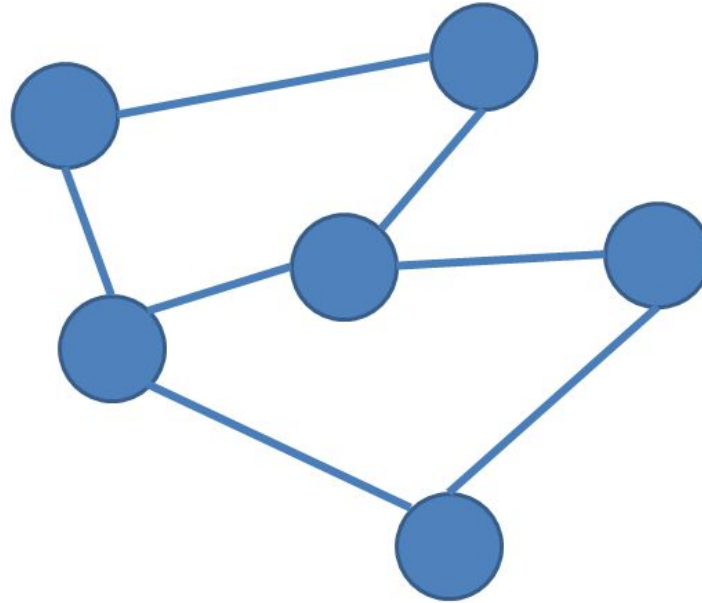
Summary

In this course we are going to focus on:

- Practical study of the data to find principles
- Mathematical models of the networks
 - Small-world model, structural balance,
- **Algorithms** (analyzing the network)
 - Communities detection, link prediction, influence maximization...
- **Applications**

Structure of the Network

Components of the Network



- **Vertices, Nodes** – objects/individuals **[V]**
- **Edges, Links** – interactions/relations **[E]**
- **Graph, Network** – the system **[G(V, E)]**

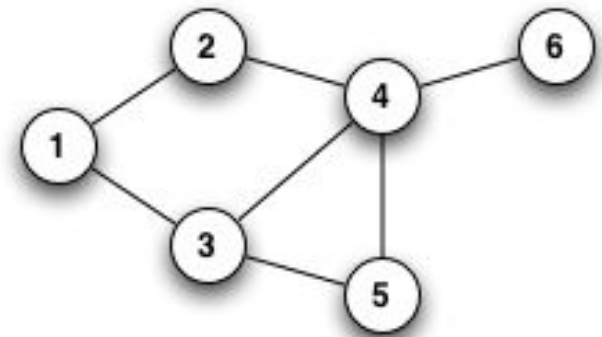
Modeling as Social Network

- Identify the domain:
 - Which problem you are trying to solve?
 - What are the nodes of the network?
 - What are the links of the network?
- .

Directed/Undirected Graphs

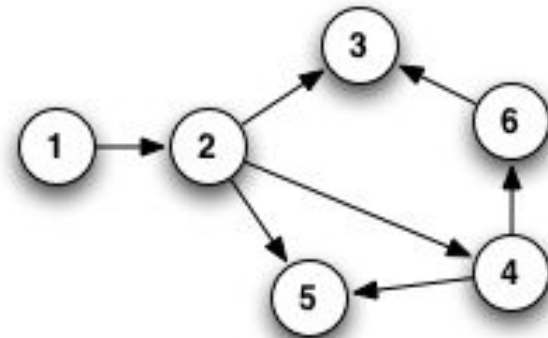
Undirected graph:

- Undirected, symmetrical edges
- Examples:
 - Friends (on Facebook)
 - Classmates



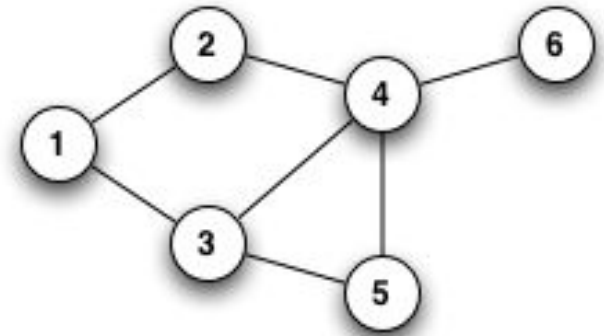
Directed graph:

- Directed edges
- Examples:
 - Followers (Instagram)
 - Phone calls



Node degree (Undirected)

Node degree (k_i) – number of edges adjacent to the node i



Example:

$$k_5 = 2, k_3 = 3$$

Average degree:

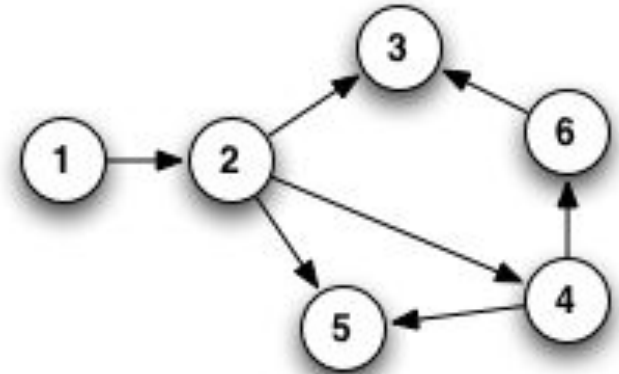
$$\langle k \rangle = 1/|V| * (k_1 + \dots + k_{|V|}) = 2|E|/|V|$$

Node degree (Directed)

In-degree (k_i^{in}) – number of edges that goes to the node

Out-degree (k_i^{out}) – number of edges that goes from the node

Total degree is a sum of in and out degrees.



Example:

$$k_5^{\text{in}} = 2, k_5^{\text{out}} = 0, k_5 = 2+0=2 \quad k_1^{\text{in}} = 0, k_1^{\text{out}} = 1, k_1 = 1$$

Avg. degree: $\langle k \rangle = |E| / |V|$, $\langle k^{\text{out}} \rangle = \langle k^{\text{in}} \rangle$

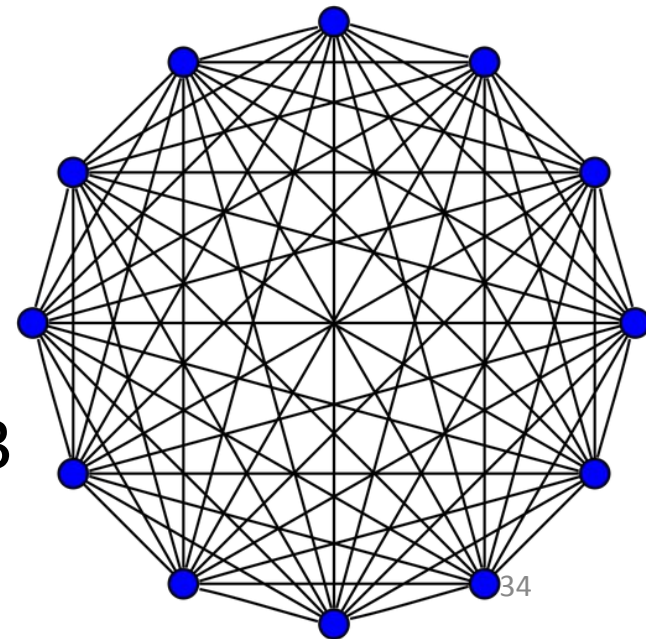
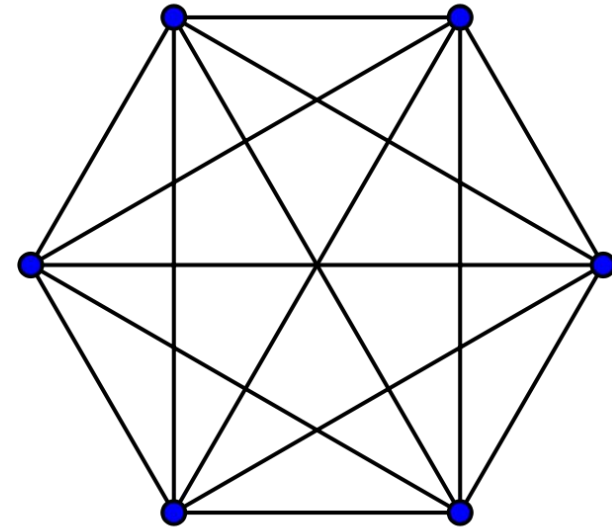
Complete Graph

The maximum number of edges in a graph of N nodes is

$$N*(N-1)/2$$

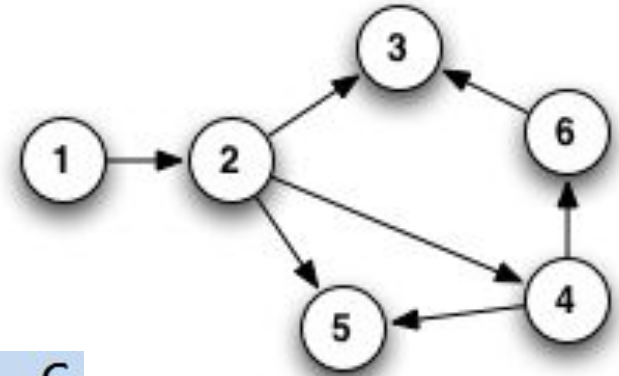
Undirected graph with maximum number of edges called **complete**

- clique is a complete subgraph
- triangle is a complete graph of size 3



Representing networks: Adjacency matrix

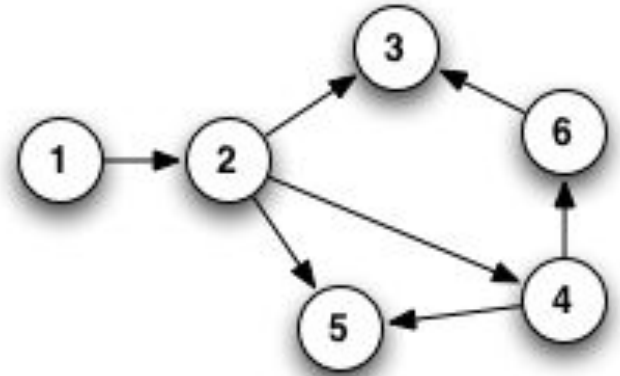
- $A_{ij} = 1$, if there is an edge (i, j)
- $A_{ij} = 0$, otherwise



	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0	0	1	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	1	1
5	0	0	0	0	0	0
6	0	0	1	0	0	0

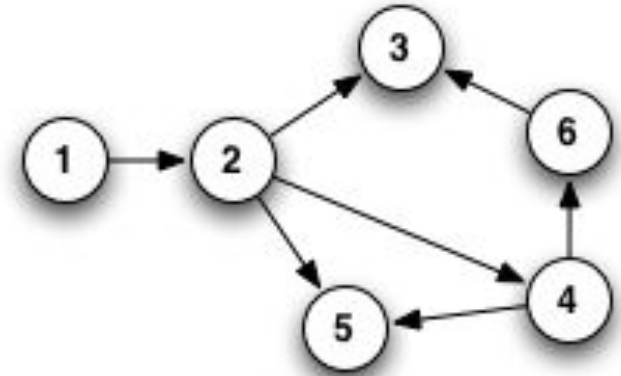
Representing networks: Edge list

- (1, 2)
- (2, 3)
- (2, 4)
- (2, 5)
- (4, 5)
- (4, 6)
- (6, 3)



Representing networks: Adjacency list

Easier for **large** and **sparse** graphs



- **1:** 2
- **2:** 3, 4, 5
- **3:**
- **4:** 5, 6
- **5:**
- **6:** 3

Social Networks are sparse

Most of the real world social networks are sparse

$$|E| \ll |E_{\max}| \quad \text{or} \quad \langle k \rangle \ll |V| - 1$$

For example, in the LinkedIn social network:

$$|V| \approx 7,000,000 \quad \langle k \rangle \approx 8.87$$

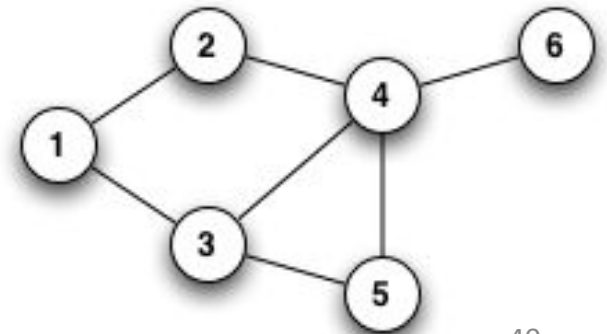
(Source: Leskovec et al., Internet Mathematics, 2009)

Edge attributes

- Weight (# messages, frequency of interaction)
- Ranking (most favorite actor, second favorite..)
- Type (friend, colleague, coauthor)
- Sign (positive/negative relationships)
- Properties depending on the other graph (number of common friends)

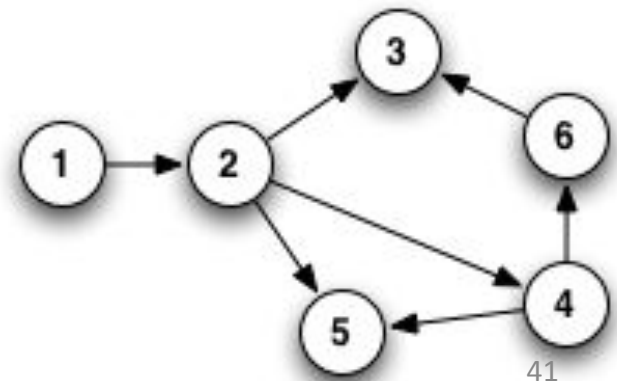
Connectivity of Undirected graphs

- **Connected graph** - any two nodes can be joined by a path (sequence of edges)
- **Disconnected graph** made out of 2 or more connected components
- **Bridge edge** – if we remove it, the graph becomes disconnected
- **Articulation node** - if we remove it, the graph becomes disconnected



Connectivity of Directed graphs

- **Strongly connected directed graph** – has a node from each node to each other node and vice-versa
- **Weakly connected directed graph** – connected if we ignore the edge directions



Quiz

For each of the examples, answer if the graph is directed/undirected and if edges are weighted or not

- Classmates –
- Facebook friends –
- Mobile phone calls –
- Twitter followers –
- Likes of Facebook –

Quiz

For each of the examples, answer if the graph is directed/undirected and if edges are weighted or not

- Classmates – undirected, weighted
- Facebook friends – undirected, non-weighted
- Mobile phone calls – directed, weighted
- Twitter followers – directed, non-weighted
- Likes of Facebook – directed, weighted

Network Properties

Key Network Properties

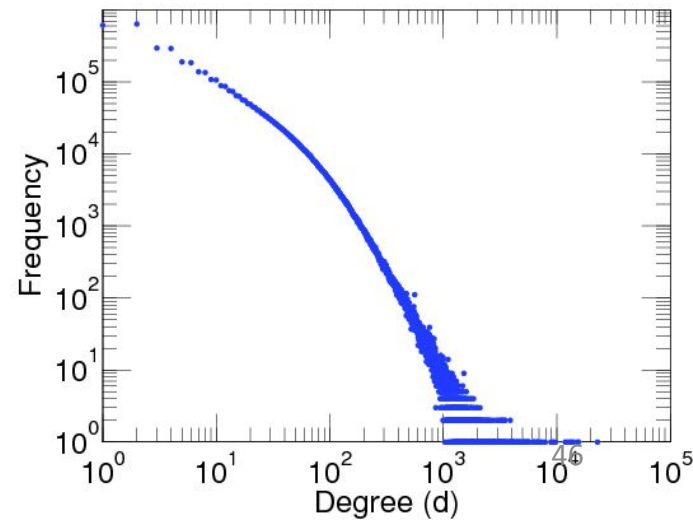
- Degree distribution $P(k)$
- Path length h
- Clustering coefficient C

Degree distribution

- $P(k)$ – probability that a randomly chosen node has a degree k

Given a graph with N nodes:

- $P(k) = N_k / N$ ($N_k = \#$ of nodes with degree k)
- Example of such distribution (LiveJournal)

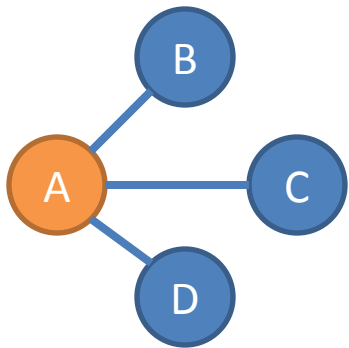


Path length

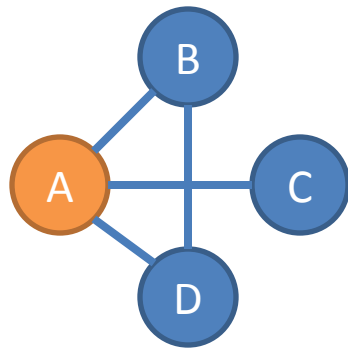
- **Path** - sequence of edges which connect a sequence of vertices which are all distinct
- **Distance** – the number of edges along the shortest path connecting two nodes
- **Diameter** – the maximal shortest path between two nodes in graph

Clustering coefficient

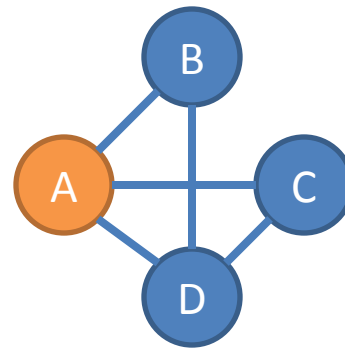
- **Clustering coefficient of a node** – fraction of the neighbors that are connected
- Node i , with degree k_i
- $C_i = 2 * (\# \text{ of edges between the neighbors}) / k_i * (k_i - 1)$
- Intuitively: # of closed triangles / # of all triangles



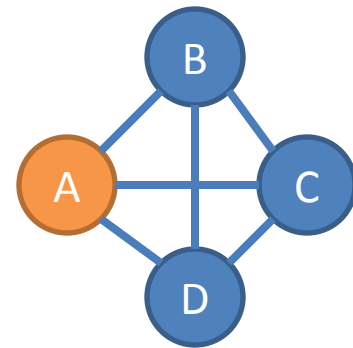
$$C_A = 0$$



$$C_A = 1/3$$



$$C_A = 2/3$$



$$C_A = 1$$

Clustering coefficient

- **Clustering coefficient of a node** – fraction of the neighbors that are connected
- Average clustering coefficient:

$$C = \frac{1}{N} \sum_i^N C_i$$



Thank you!
Questions?